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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

WATER PRESSURES ON DAMS DURING EARTHQUAKES

By H. M. Westergaard,[1] M. Am. Soc. C. E.

Synopsis

Formulas are derived for the changes of water pressures during an earthquake for the case of a straight dam with a vertical up-stream face. The vibrations in the earthquake are assumed horizontal in a direction perpendicular to the dam. A formula is obtained involving a series of sines (Equation (31)). This series, however, may be replaced with satisfactory approximation by a simpler formula (Equation (49)). The results are also expressed in the following manner: The pressures are the same as if a certain body of water were forced to move back and forth with the dam while the remainder of the reservoir is left inactive; Fig. 3 shows approximately the shape of this body of water. The influence of the dynamic action of the water is found to be neither excessively large nor negligible.

Introductory Remarks Concerning Stresses in Dams During Earthquakes

Two causes account for added stresses in a concrete dam during an earthquake: First, the accelerations of the mass of the dam; and, second, the changes of water pressures. If no fault crosses the site of the dam, one may assume for the purpose of determining the accelerations that all points of the foundation have the same displacement, velocity, and acceleration at the same time, because the wave lengths of the main vibrations in the crust of the earth are so great that the differences of accelerations and the tiltings may be considered negligible.

Note.—Written discussion on this paper will be closed in February, 1932, *Proceedings*

[1] Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.; Cons. Engr., U. S. Bureau of Reclamation.

For a straight concrete gravity dam with triangular cross-section, with height, H, and base, B, and with a modulus of elasticity of the concrete of 2 000 000 lb. per sq. in., the period of free vibrations (with the reservoir empty) would be approximately $T_0 = \frac{H^2}{2000\,B}\,\frac{\text{sec.}}{\text{ft.}}$ if the base were rigid.[2]

With $B = 0.8H$, the formula gives $T_0 = \frac{H}{1600\ \text{ft.}}$ sec.; for example, $T_0 = \frac{1}{8}$ sec. when $H = 200$ ft. On account of the elasticity of the base the actual period is longer, but remains a quantity of the same order. In an arch dam the stiffness of the arches reduces the period, as compared with that given by the formula.

Hence, since the period of free vibrations of the dam, T_0, is usually a fraction of a second, while the period of vibration, T, of the earthquake may be assumed to be not less than a second[3] (1.35 sec. measured during the great earthquake in Japan in 1923[4]), resonance—with vibrations of the structure becoming large compared with those of the foundation—need not be expected ordinarily, although the possibility of part resonance may well be investigated in some cases of high dams. When the ratio, $\frac{T_0}{T}$, is small, all points of the dam may be assumed at each time to have the same acceleration as the base. If the maximum horizontal acceleration is $a = \alpha\, g$, in which g is the acceleration due to gravity, the ratio, α, is used as a measure of the intensity of the earthquake. The value, $\alpha = 0.1$, appears to be a reasonable one to use in investigating dams founded on rock in earthquake regions in the United States.

The problem of the dynamic action of the mass of the dam is transformed into an equivalent problem of statics by introducing the inertia forces, expressed as mass times the acceleration reversed (d'Alembert's principle): that is, on each part of the dam weighing 1 lb. there is introduced a horizontal inertia force, opposite the direction of the acceleration and equal to $\frac{1\ \text{lb}}{g} \times \alpha\, g = \alpha$ lb. The vertical accelerations, which by the nature of the phenomenon are smaller, either may be treated similarly or may possibly be ignored. If they are ignored, the combined effect of the dead weight and the inertia forces may be described as follows: Each gravity force of 1 lb. is changed into an inclined force, which has a vertical component equal to 1 lb., and a slope relative to the vertical equal to α. If α is not larger than, say, 0.1, the effect is nearly the same as though the dam were tilted so that a line that was horizontal will have a slope, α. The determination of the corresponding stresses involves no new difficulties that have not already been encountered in dealing with static loads.

[2] See, for example, S. Timoshenko and J. M. Lessells, "Applied Elasticity," 1925, p. 323.

[3] Handbuch der Physik, Vol. 6, 1928, p. 596. The short periods occur only when the center of the earthquake is close.

[4] "Effect of Earth Shocks on Structures," M. de Bussy, *Proceedings*, Am. Soc. C. E., May, 1928, Papers and Discussions, p. 1449.

When the water pressures are known, their influence on the stresses may be computed by the methods applying to static loads. The problem that needs particular attention, therefore, is that of the changes of the water pressures during the earthquake.

Notation

The following notation is adopted:

x, y, z = rectangular co-ordinates x and y as shown in Fig. 1; the axis of x is along the undisturbed surface of the water, is directed up stream, and is perpendicular to the plane up-stream face of the dam; the axis of y is vertical downward.
t = time.
h = depth of reservoir.
g = acceleration due to gravity. (See Equations (10).)
w = weight of water per unit of volume. (See Equations (10).)
k = modulus of elasticity of volume of the water (pressure divided by reduction of volume per unit of volume). (See Equations (10).)
ξ and η = displacements of a particle of water in the directions of x and y, respectively.
ξ_0 = value of ξ at up-stream face of the dam.
σ = stress in the water due to the dynamic action, positive as tension (to be combined with the hydrostatic pressure existing beforehand).
α = ratio measuring the intensity of the earthquake = maximum horizontal component of the acceleration of the foundation divided by g.
T = period of horizontal vibrations of the foundation.
r, v = amplitude and maximum velocity of horizontal vibrations of the foundation.
v_s = velocity of sound in water.
l = length of waves of expansion in the water when the period is T.
c_n = value defined by Equations (17), (23), and (32) ($n = 1, 3, 5 \ldots$).
p = maximum pressure of the water on the dam at the depth, y, due to the dynamic action.
p_0 = value of p at the bottom ($y = h$).
Q = horizontal shear produced in a straight gravity dam at the depth, y, by the pressures, p.
Q_0 = value of Q at the bottom ($y = h$).
M = bending moment produced in a straight gravity dam at the depth, y, by the pressures, p.
M_0 = value of M at the bottom ($y = h$).
C, K = coefficients in approximate formulas (Equations (40) to (48).
b, b' = dimensions shown in Fig. 3.

Equations of Motion of the Water

Since the motions involved are small, the relatively simple equations may be used, that apply to sound in liquids.[5] These equations may be interpreted in terms of the theory of elasticity of solids as equations of elasticity without shearing stresses.

The forces acting on an element of volume, $dx\ dy\ dz$, are expressed in terms of the tensile stress, σ, which must be superimposed on the hydrostatic

[5] "Hydrodynamics," by Horace Lamb, Fifth Edition, 1924, Chapter X.

pressure existing before the disturbance. The force on the face with the abscissa, $x + dx$, exceeds that on the face with the abscissa, x, by the amount, $\frac{\partial \sigma}{\partial x}\, dx$ per unit of area, or, by a total amount of $\frac{\partial \sigma}{\partial x}\, dx\, dy\, dz$. The mass of the element is $\frac{w}{g}\, dx\, dy\, dz$, and its acceleration is $\frac{\partial^2 \xi}{\partial t^2}$ (ξ being the displacement in the direction of x). Equating the force to the mass times the acceleration one finds the first of the following two equations:

$$\frac{\partial \sigma}{\partial x} = \frac{w}{g} \frac{\partial^2 \xi}{\partial t^2} \quad \ldots\ldots (1)$$

$$\frac{\partial \sigma}{\partial y} = \frac{w}{g} \frac{\partial^2 \eta}{\partial t^2} \quad \ldots\ldots (2)$$

Equation (2) refers to the motion in the direction of y, and applies by analogy; a third analogous equation, for the direction of z, need not be written, since only plane motions, parallel to the xy-plane, will be considered.

The stress, σ, in Equations (1) and (2), has the relation,

$$\sigma = k\, \epsilon \quad \ldots\ldots (3)$$

to the increase of volume per unit of volume, ϵ, that it produces. The constant, k, is the modulus of elasticity of volume. One may write,

$$\epsilon = \frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} \quad \ldots\ldots (4)$$

Consequently,

$$\sigma = k \left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} \right) \quad \ldots\ldots (5)$$

The motion is governed by Equations (1), (2), and (5). A comparison with the general equations of hydrodynamics for compressible fluids[8] shows that Equations (1), (2), and (5) may be assumed to apply when ξ and η are small, and when, in addition, the ratios of the products, $\frac{\partial \xi}{\partial t} \times \frac{\partial \sigma}{\partial x}$ and $\frac{\partial \eta}{\partial t} \times \frac{\partial \sigma}{\partial y}$ to $\frac{\partial \sigma}{\partial t}$, are negligible. In the solutions given in Equations (15), (16), and (20), these ratios are found to be of the order of the amplitude of the motions divided by the depth of the reservoir.

Preliminary Study, Assuming Horizontal Motion Only

The fact that the water, when pressed by the dam, may escape vertically, is important. A preliminary study in which this vertical motion is assumed to be barred, will nevertheless throw some light on the problem.

With $\eta = 0$, Equations (1) and (5) give:

$$k \frac{\partial^2 \xi}{\partial x^2} = \frac{w}{g} \frac{\partial^2 \xi}{\partial t^2} \quad \ldots\ldots (6)$$

[8] See, for example, Handbuch der Physik, Vol. VII, 1927, p. 291.

Assume that the dam moves during the earthquake in a simple harmonic motion with period, T, and a maximum acceleration, αg, occurring when $t = 0, T, 2T$, etc. This motion is expressed by the equation,

$$\xi_0 = -\frac{\alpha g T^2}{4\pi^2} \cos \frac{2\pi t}{T} \quad \ldots\ldots(7)$$

The requirement, $\xi_{x=0} = \xi_0$, and Equation (6) are satisfied by the solution,[7]

$$\xi = -\frac{\alpha g T^2}{4\pi^2}\left[\beta \cos \frac{2\pi}{T}\left(t - \frac{x}{v_s}\right) + (1-\beta) \cos \frac{2\pi}{T}\left(t + \frac{x}{v_s}\right)\right] \quad \ldots\ldots(8)$$

in which, β is a constant, and,

$$v_s = \sqrt{\frac{gk}{w}} \quad \ldots\ldots(9)$$

The quantity, v_s, is the velocity of the waves of expansion in the water; that is, the velocity of sound (on account of the low frequency, these waves, of course, would not be audible as sound). The values,

$$\left.\begin{aligned} k &= 300\,000 \frac{\text{lb.}}{\text{in.}^2} = 21\,600 \frac{\text{ton}}{\text{ft.}^2} \\ g &= 32.2 \frac{\text{ft.}}{\text{sec.}^2} \\ w &= 0.03125 \frac{\text{ton}}{\text{ft.}^2} \end{aligned}\right\} \quad \ldots\ldots(10)$$

give $v_s = 4\,718$ ft. per sec. The value, $v_s = 4\,714$ ft. per sec., was measured by Wertheim in the Seine River at the temperature of 15° cent.[8]

The term in Equation (8) containing the factor, β, in front of the cosine represents waves moving away from the dam, while that containing the factor, $1 - \beta$, represents waves moving toward the dam.

Equations (8) and (5) give for $x = 0$ (that is, at the up-stream face of the dam):

$$\sigma = (1 - 2\beta) \frac{\alpha T}{2\pi} \sqrt{gkw} \sin \frac{2\pi t}{T} \quad \ldots\ldots(11)$$

When $\beta > \frac{1}{2}$, that is, when the waves moving away from the dam predominate over those moving toward it, Equation (11) shows that the maximum pressure on the dam, that is, $(-\sigma)_{\max.}$, occurs at the times $t = \frac{T}{4}, \frac{5T}{4}, \frac{9T}{4}, \ldots\ldots$, and is equal to,

$$p = (2\beta - 1) \frac{\alpha T}{2\pi} \sqrt{gkw} \quad \ldots\ldots(12)$$

It is noted that the greatest acceleration of the dam, and, consequently, the greatest inertia forces due to the moving mass of the dam, occur when $t = 0, T, 2T, \ldots$. The stresses in the dam due to the pressures, p, and the

[7] "Hydrodynamics", by Horace Lamb, Fifth Edition, 1924, p. 452.
[8] Smithsonian Physical Tables, Seventh Edition, 1921, p. 147.

inertia forces, therefore, would have to be combined in the manner of two alternating currents with a difference of phase of one-quarter of the period.

Since, according to Equation (7), the maximum velocity of the dam is,

$$v = \frac{\alpha g t}{2\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Equation (12) may be rewritten, by the use of Equation (9), in the form,

$$p = (2\beta - 1)\frac{v}{v_s}k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

With $\beta = 1$, that is, when the waves move away from the dam only, Equation (12) (or, if one prefers, Equation (14)), with the numerical constants stated in Equations (10), and with $\alpha = 0.1$ and $T = \frac{4}{3}$ sec., gives $p = 3.13$ tons per sq. ft. On account of the freedom of the water to escape vertically the pressures are actually much less, and are not distributed uniformly through the depth of the reservoir. A solution which is to be used must show the pressures in the water to be zero at the top.

Solution Considering Horizontal and Vertical Motions of the Water

A solution is desired which satisfies Equations (1), (2), and (5), and, in addition, the following conditions at the boundary:

(1) $\sigma = 0$, when $y = 0$;

(2) $\eta = 0$, when $y = h$;

(3) $\xi = -\frac{\alpha g T^2}{4\pi^2}\cos\frac{2\pi t}{T}$, when $x = 0$; and,

(4) σ converges toward 0 when x becomes large.

In connection with Condition (1), it is noted that when the values of η are small, it makes no appreciable difference whether one specifies $\sigma = 0$ at the undisturbed or the disturbed surface.

The following solution will be shown to satisfy all these conditions:

$$\xi = -\frac{\alpha g T^2}{\pi^3}\cos\frac{2\pi t}{T}\sum_{1,3,5,\ldots}^{n}\frac{1}{n}e^{-qn}\sin\frac{n\pi y}{2h} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

and,

$$\eta = \frac{\alpha g T^2}{\pi^3}\cos\frac{2\pi t}{T}\sum_{1,3,5,\ldots}^{n}\frac{1}{nc_n}e^{-qn}\cos\frac{n\pi y}{2h} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which,

$$c_n = \sqrt{1 - \frac{16wh^2}{n^2 g k T^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

and,

$$q_n = \frac{n\pi c_n x}{2h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Equation (17) gives,

$$\frac{1}{c_n} - c_n = \frac{16 w h^2}{n^2 c_n g k T^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Then Equations (5), (15), (16), and (19) lead to the value of the stress,

$$\sigma = -\frac{8\alpha w h}{\pi^2} \cos\frac{2\pi t}{T} \sum_{1,3,5,\ldots}^{n} \frac{1}{n^2 c_n} e^{-q_n} \sin\frac{n\pi y}{2h} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

That the solution represented by Equations (15), (16), and (20) satisfies Equations (1) and (2) at all points is verified easily by performing the required differentiations with respect to t, x, and y. Equation (5) is satisfied because it was used in deriving Equation (20) from Equations (15) and (16). Furthermore, inspection of the expressions for η and σ shows immediately that Conditions (1), (2), and (4) are satisfied. That Condition (3), for $x = 0$, is satisfied, is seen by comparing Equation (15) for $x = 0$ with the expression for $\frac{\pi}{4}$, in terms of a Fourier series,[9] applicable when $0 < u < \pi$,

$$\frac{\pi}{4} = \sin u + \frac{1}{3}\sin 3u + \frac{1}{5}\sin 5u + \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

In the present application, $u = \frac{\pi y}{2h}$.

Thus, all the specified conditions are satisfied. The applicability of the equations depends, however, on an additional condition: the displacements defined by the solution, as well as their derivatives with respect to x and y,[10] must be small. It is necessary then to examine the possibilities of large displacements. One possibility is identified with resonance in the water; another will be investigated by determining the shape of the wave formed by the surface of the water and the vertical motions along the up-stream face of the dam.

Possibility of Resonance in the Water

The waves of expansion (or waves of sound) represented by Equations (8) and (9) have the wave length,

$$l = v_s T = T\sqrt{\frac{gk}{w}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

By use of this value Equation (17) may be rewritten in the form,

$$c_n = \sqrt{1 - \frac{16h^2}{n^2 l^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

[9] See, for example, "Short Table of Integrals", by B. O. Pierce, Second Edition, 1910, p. 95, Formula 808.

[10] Compare the remark following Equation (5).

With $n = 1$, one finds $c_1 = 0$ if $h = \frac{l}{4}$, that is, if the depth of the reservoir is one-fourth of a wave length. This case is analogous to that of an organ pipe open at the top; one may expect the water to vibrate like the air in the pipe. That resonance occurs, is shown by Equations (16) and (20); they give infinite values of η and σ in this case. If h approaches the value, $\frac{l}{4}$, the equations lead to large values of η and σ.

With the period, $T = 1$ sec., $\frac{l}{4}$ becomes $\frac{1}{4} \times 4\,718 \frac{\text{ft.}}{\text{sec.}} \times 1 \text{ sec.} = 1\,180$ ft.

Since the main vibrations in the earthquake are not likely to have a period less than 1 sec., this form of resonance need not be expected in any project now contemplated.

Shape of the Wave and Vertical Motions at Up-Stream Face of Dam

In actual cases the values of c_n do not differ greatly from 1. In examining the vertical displacements it will be permissible, therefore, to write $c_n = 1$. In terms of the maximum displacement of the dam according to Condition (3) (stated shortly before Equation (15)),

$$r = \frac{\alpha g T^2}{4\pi^2} \qquad (24)$$

Equation (16) gives then the following expressions for η at the top ($y = 0$) and at the up-stream face ($x = 0$) when $t = 0, T, 2T, \ldots$:

$$\eta_{y=0} = \frac{4r}{\pi} \sum_{1,3,5,\ldots}^{n} \frac{1}{n} e^{-\frac{n\pi x}{2h}} \qquad (25)$$

and,

$$\eta_{x=0} = \frac{4r}{\pi} \sum_{1,3,5,\ldots}^{n} \frac{1}{n} \cos \frac{n\pi y}{2h} \qquad (26)$$

By use of the formulas,[11]

$$\left.\begin{aligned} \tfrac{1}{2} \log_e \frac{1+u}{1-u} &= u + \frac{u^3}{3} + \frac{u^5}{5} + \frac{u^7}{7} + \ldots \ldots \\ \text{and,} \\ -\tfrac{1}{2} \log_e \tan \frac{U}{2} &= \cos U + \frac{1}{3} \cos 3U + \frac{1}{5} \cos 5U + \ldots \end{aligned}\right\} \qquad (27)$$

[11] "Short Table of Integrals", by B. O. Pierce, Second Edition, 1910, pp. 82, 94, and 96, Formulas 681, 797, 813, and 814.

with the substitutions, $u = e^{-\frac{\pi x}{2h}}$, $U = \frac{\pi y}{2h}$, one finds,

$$\eta_{y=0} = \frac{2r}{\pi} \log_e \frac{1 + e^{-\frac{\pi x}{2h}}}{1 - e^{-\frac{\pi x}{2h}}} = -\frac{2r}{\pi} \log_e \tanh \frac{\pi x}{4h} \quad \ldots\ldots\ldots (28)$$

and,

$$\eta_{x=0} = -\frac{2r}{\pi} \log_e \tan \frac{\pi y}{4h} \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

These formulas give $\eta = \infty$ at the point, $x = 0$, $y = 0$; that is, the solution in Equations (15), (16), and (20) has a singularity (infinite values) at this point, and the condition that the displacements and their first derivatives must remain small, is not satisfied at this point and in the neighborhood of this point. The solution, nevertheless, may very well be adequate (except at and near the singularity), if only the region of large values of η can be shown to be small.

When $x = 0.1h$, one finds $\tanh \frac{\pi x}{4h} = \tanh 0.07854 = 0.07838$. Likewise, when $y = 0.1h$, one finds $\tan \frac{\pi y}{4h} = \tan 0.07854 = 0.07870$. For these and smaller values of x and y, one may accordingly replace Equations (28) and (29) by the simpler approximate formula,

$$\eta_{y=0} \text{ or } \eta_{x=0} = -\frac{2r}{\pi} \log_e \frac{\pi (x \text{ or } y)}{4h} = \left(1.466 \log_{10} \frac{h}{x \text{ or } y} + 0.1538\right) r \ldots (30)$$

Assume, for example, that $\alpha = 0.1$, $T = \frac{4}{3}$ sec.; then Equation (24) gives the amplitude of the motion of the dam, $r = 0.1450$ ft. With $h = 200$ ft., Equation (30) gives the values shown in Table 1.

TABLE 1.—VALUES OF η BY EQUATION (30) FOR $h = 200$ FEET

At distance, x or y (in feet)	0.1	1	5	10	20
Movement, $\eta_{y=0}$, or $\eta_{x=0}$ (in feet)	0.724	0.511	0.363	0.299	0.235

These computations show that the region about the point, $x = 0$, $y = 0$, within which the displacements become large is so small that the discrepancies introduced by the singularity at this point can have only a negligible influence on the solution as a whole. The solution in Equations (15), (16) and (20) has no other singularities (points at which infinite values are found), and therefore may be accepted as applying to the case.

Pressures on the Dam

According to Equation (20), the maximum pressures, p ($= -\sigma$ for $x = 0$), of the water on the dam occur when $t = 0, T, 2T$, etc. One finds,

$$p = \frac{8\alpha w h}{\pi^2} \sum_{1,3,5,\ldots}^{n} \frac{1}{n^2 c_n} \sin \frac{n\pi y}{2h} \quad \ldots\ldots(31)$$

in which, as before, according to Equation (17), and with the numerical constants stated in Equations (10):

$$c_n = \sqrt{1 - \frac{16\, w h^2}{n^2 g k T^2}} = \sqrt{1 - \frac{0.71889}{n^2} \left(\frac{h \text{ sec.}}{1\,000\, T \text{ ft.}}\right)^2} \quad \ldots\ldots(32)$$

The greatest value of the pressure, p, occurs at the bottom of the reservoir ($y = h$), and is,

$$p_o = \frac{8\alpha w h}{\pi^2} \sum_{1,3,5,\ldots}^{n} \frac{(-1)^{\frac{n-1}{2}}}{n^2 c_n} \quad \ldots\ldots(33)$$

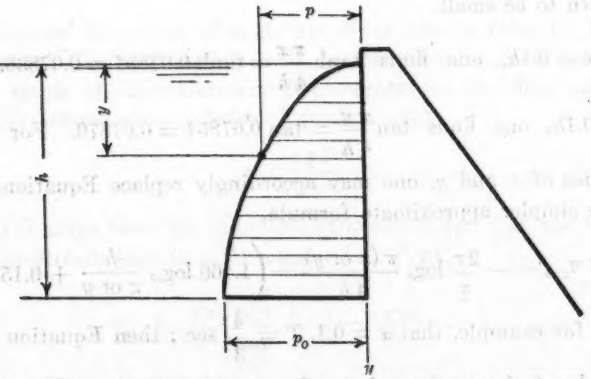

Fig. 1.—Pressures on Dam Due to Dynamic Action of the Water

Fig. 1 shows the general shape of the diagram of the pressures, p. According to Equation (31), the derivative, $\frac{dp}{dy}$, is infinite when $y = 0$, and zero when $y = h$; that is, the curve for p has a horizontal tangent at the top and a vertical tangent at the bottom. Items 1 and 2 of Table 2 contain some numerical values of the ratio, $\frac{p_o}{\alpha h}$, and of p_o, computed from Equations (32) and (33). The ratio, $\frac{p_o}{\alpha h}$, is seen to vary only slowly with h. In the case of $h = 600$ ft., $T = \frac{4}{3}$ sec. (or $h = 200$ ft., $T = \frac{4}{9}$ sec.), the pressure at the depth, $y = \frac{h}{2}$, was found to be $p = 0.814 p_o$.

TABLE 2.—NUMERICAL RESULTS CORRESPONDING TO THE PERIOD, $T = \frac{4}{3}$ SECONDS

(The maximum horizontal acceleration equals $\alpha\, g$. When T is varied, the coefficients stated remain constant if h is changed by the same ratio; for example, the coefficients stated for $h = 800$ ft., $T = \frac{4}{3}$ sec., apply to the cases, $h = 600$ ft., $T = 1$ sec., and $h = 200$ ft., $T = \frac{1}{3}$ sec.; the coefficients stated for $h = 200$ ft., $T = \frac{4}{3}$ sec., apply to the case, $h = 600$ ft., $T = 4$ sec.)

Item	Depth of reservoir, h	Ft.	Small	200	600	800
1	Coefficient of pressure, $\frac{p_0}{\alpha h}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.023202	0.023407	0.025254	0.027257
2	Pressure at bottom, p_0, when $\alpha = 0.1$	$\frac{\text{ton}}{\text{ft.}^2}$	.....	0.468	1.515	2.181
3	Coefficient of shear, $\frac{Q_0}{\alpha h^2}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.016961	0.017094	0.018286	0.019576
4	Shear at bottom, Q_0, when $\alpha = 0.1$ ..	$\frac{\text{ton}}{\text{ft.}}$	.....	68.4	658.3	1 253
5	Coefficient of moment, $\frac{M_0}{\alpha h^3}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.0068089	0.0068576	0.0072947	0.0077666
6	Moment at bottom, M_0, when $\alpha = 0.1$	$\frac{\text{ft.-ton}}{\text{ft.}}$	.....	5 486	157 600	397 609
7	$C = \frac{15\, M_0}{4\, \alpha\, h^3}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.025533	0.025716	0.027355	0.029125
8	$K = C\sqrt{1 - 0.72\left(\frac{h \text{ sec.}}{1\,000\, T \text{ ft.}}\right)^2}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.025533	0.025507	0.025282	0.025068
9	Approximate coefficients computed from Equations (40), (41), (42), (48), and (49).... $\frac{p_0}{\alpha h}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.0255	0.0257	0.0276	0.0296
10	$\frac{Q_0}{\alpha h^2}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.0170	0.0171	0.0184	0.0198
11	$\frac{M_0}{\alpha h^3}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.00680	0.00686	0.00736	0.00790

SHEARS AND MOMENTS IN THE DAM

It is desirable to express the horizontal shearing forces and the bending moments produced by the loads, p, in various horizontal sections of the dam when the dam functions as a cantilever (gravity dam). The shears are stated as force per unit of width of the cantilever, for example, in tons per foot, and the moments are stated as moment per unit of width, for example, in foot-tons per foot (or simply in tons). The shears, Q, at the depth, y, and Q_0, at the depth, h, and the moments, M, at the depth y, and M_0, at the depth, h, are obtained from p in Equation (31) by the integrations,

$$Q = \int_0^y p\, dy;\ Q_0 = \int_0^h p\, dy;\ M = \int_0^y Q\, dy;\ M_0 = \int_0^h Q\, dy \ldots\ldots (34)$$

With the notation,

$$q = \sum_{1,\,3,\,5,\,..}^{n} \frac{1}{n^3 c_n} \qquad (35)$$

one finds by this process:

$$Q = \frac{16\,\alpha\,w\,h^2}{\pi^3}\left(q - \sum_{1,\,3,\,5,\,..}^{n} \frac{1}{n^3 c_n} \cos\frac{n\pi y}{2h}\right) \qquad (36)$$

$$Q_0 = \frac{16}{\pi^3}\,\alpha\,w\,h^2 q \qquad (37)$$

$$M = \alpha\,w\,h^3\left(\frac{16q}{\pi^3}\,\frac{y}{h} - \frac{32}{\pi^4}\sum_{1,\,3,\,5,\,..}^{n} \frac{1}{n^4 c_n} \sin\frac{n\pi y}{2h}\right) \qquad (38)$$

and,

$$M_0 = \alpha\,w\,h^3\left(\frac{16q}{\pi^3} - \frac{32}{\pi^4}\sum_{1,\,3,\,5,\,..}^{n} \frac{(-1)^{\frac{n-1}{2}}}{n^4 c_n}\right) \qquad (39)$$

Numerical values of the ratios, $\frac{Q_0}{\alpha h^2}$ and $\frac{M_0}{\alpha h^3}$, and of Q_0 and M_0, computed from Equations (35), (37), and (39), are stated in Table 2 (see, Items, 3, 4, 5, and 6).

Derivation of Approximate Formulas

For the purpose of approximate computations one might replace the diagram of pressures in Fig. 1 by a quadrant of an ellipse (see Equation (50) derived subsequently); the ellipse becomes a circle when drawn to some particular scale for p. A more satisfactory approximate representation is obtained, however, by replacing the curve in Fig. 1 by a parabola with vertical axis, as shown in Fig. 2, even if this parabola has a sloping tangent at the bottom.

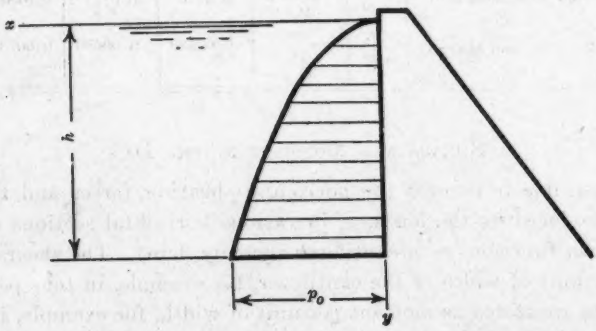

Fig. 2.—Approximate Distribution of Pressures.

The parabola leads to the following simple formulas, in which, C is a coefficient depending on the ratio of h to T:

$$p = C\,\alpha\,\sqrt{hy}\;;\; p_0 = C\,\alpha\,h \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

$$Q = \frac{2}{3}\,C\,\alpha\,y\,\sqrt{hy}\;;\; Q_0 = \frac{2}{3}\,C\,\alpha h^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

and,

$$M = \frac{4}{15}\,C\,\alpha y^2\,\sqrt{hy}\;;\; M_0 = \frac{4}{15}\,C\,\alpha h^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

The coefficient, C, might be determined so that p_0 will have the value computed from Equation (33), but one obtains a better representation of the diagram of pressures as a whole, by determining C so that the moment, M_0, will have the value computed from Equation (39). In terms of a known M_0, one finds then,

$$C = \frac{15\,M_0}{4\,\alpha h^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Table 2 (Item 7) shows some numerical values of C. It is noted that the dimension of C is force per unit of volume; C multiplied by αh gives the pressure as force per unit of area.

The coefficient, C, varies slowly within the range considered. For quick estimates one may very well assume some constant value of C; for example, with $T \geq \frac{4}{3}$ sec.:

When $h < 310$ ft.:

$$C = 0.026\ \frac{\text{ton}}{\text{ft.}^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

when 310 ft. $< h <$ 540 ft.:

$$C = 0.027\ \frac{\text{ton}}{\text{ft.}^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

and when 540 ft. $< h <$ 680 ft.:

$$C = 0.028\ \frac{\text{ton}}{\text{ft.}^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

It is a simple matter, however, to express C by a fairly convenient formula which will apply approximately within the entire range. The main source of variation of C is the constant, c_1, defined by Equation (32) with $n = 1$; c_1 appears in the denominator of the first term of each of the sums in Equations (35) and (39). It is reasonable, therefore, to try out an approximate formula of the type, $C = \frac{K}{c_1}$, in which, K is a constant. The formula is written,

$$C = \frac{K}{\sqrt{1 - 0.72\left(\frac{h\ \text{sec.}}{1\,000\,T\ \text{ft.}}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

The values of K shown in Table 2 (Item 8) were computed from this formula by using the values of C in Item 7. Inspection of these results shows that

one may adopt the value, $K = 0.0255$ ton-ft.$^{-3}$. It is concluded that the coefficient, C, in Equations (40), (41), and (42), is expressed with satisfactory approximation by the formula,

$$C = \frac{0.0255 \text{ ton-ft.}^{-3}}{\sqrt{1 - 0.72\left(\frac{h \text{ sec.}}{1\,000\,T \text{ ft.}}\right)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Resulting Approximate Formula for the Water Pressures

The following approximate formula for the maximum water pressures, to be added to the hydrostatic pressures existing before the earthquake, represents the main conclusion from this study; it is obtained directly from Equations (40) and (48):

$$p = \frac{0.0255 \text{ ton-ft.}^{-3} \alpha \sqrt{hy}}{\sqrt{1 - 0.72\left(\frac{h \text{ sec.}}{1\,000\,T \text{ ft.}}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

For quick computations, however, Equations (40), (44), (45), and (46) may be used. Values of the coefficients, $\frac{p_0}{\alpha h}$, $\frac{Q_0}{\alpha h^2}$, and $\frac{M_0}{\alpha h^3}$, obtained by use of Equation (49), are stated as Items 9, 10, and 11 in Table 2. A comparison with Items 1, 3, and 5, respectively, shows that the approximation is satisfactory.

By using a quadrant of an ellipse (as suggested under "Derivation of Approximate Formulas") instead of the parabola to represent the pressures, one obtains the following approximate formula by a process similar to that leading to Equation (49):

$$p = \frac{0.0204 \text{ ton-ft.}^{-3} \alpha \sqrt{y\,(2h - y)}}{\sqrt{1 - 0.72\left(\frac{h \text{ sec.}}{1\,000\,T \text{ ft.}}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

This formula gives exactly the same values of M_0 as Equation (49), but gives too small values of p_0 and of the total force, Q_0. Equation (49) gives slightly too large values of p_0 and Q_0, and is preferable.

It is noted that the pressures, p, occur when $t = 0$, T, $2T$, etc., that is, (see Condition (3)), when the dam (during the vibration) is in the extreme position down stream. The stresses due to these water pressures, therefore, must be added to the maximum stresses due to the accelerations of the mass of the dam iself.

When, during the vibration, the dam is in the extreme position up stream, the pressures, p, are reversed and become tensions. These tensions, like the pressures in the extreme position down stream, are to be combined with the hydrostatic pressures existing beforehand. Inspection of the numerical values in Table 2 shows that when the pressure of the atmosphere is included, there is no prospect of the resultant stress in the water becoming either a tension or a very small compression.

Applicability of Formulas to Cases Other than Those Assumed

The formulas were derived under the assumption that there is no overflow. In case of overflow over a spillway it is suggested that Equation (49) be used with the depths, h and y, measured from the plane surface of the reservoir at some distance behind the dam. The parabolic diagram of pressures is then used as before, only the top is cut off.

While the formulas were derived under the assumption that the up-stream face is a vertical plane, they will furnish an indication of the pressures which may be expected in other cases, for example, in the case of arch dams.

The Body of Water Which May Be Considered to Move with the Dam

To visualize the dynamic action of the water on the dam, one may think of a certain body of water as moving with the dam while the remainder of the reservoir remains inactive. For this purpose the body of water moving with the dam may be imagined as frozen into horizontal layers of ice (the expansion by freezing being ignored), while the remainder of the reservoir is emptied. The layers of ice are imagined to support one another by vertical forces only, without friction between the layers; the layers are attached firmly to the dam, so that the dam will exert the horizontal forces necessary to move them back and forth with it. The advantage of picturing the action in this manner is that the forces exerted on the up-stream face of the dam will then be represented as inertia forces like those due to the moving mass of the dam itself.

The shape of the body of water or ice thus considered to move with the dam must be determined so that the inertia forces become equal to the pressures actually exerted by the water due to the dynamic action. With b denoting the dimension of this body in the direction of x at the depth, y, the corresponding mass per unit of area of the up-stream face of the dam becomes $\frac{b\,w}{g}$, and the corresponding inertia force when the acceleration is $\alpha\,g$ becomes $\frac{b\,w}{g} \times \alpha\,g = \alpha\,w\,b$, which must be equal to p. One finds then:

$$b = \frac{p}{\alpha\,w} \qquad (51)$$

That is, the shape of this body of water or ice is the same as that of the diagram for p when the pressures are laid off to a proper scale.

It will be sufficient for the present purpose to determine b by one of the approximate expressions for p; for example (compare Equations (40), (44), (45), and (46)),

$$p = 0.02734 \text{ ton-ft.}^{-3}\,\alpha\,\sqrt{h\,y} \qquad (52)$$

which gives, with $w = 0.03125$ ton-ft.$^{-3}$,

$$b = \frac{7}{8}\sqrt{h\,y} \qquad (53)$$

Fig. 3 shows the body of water or ice defined by Equation (53).

Instead of the body of water, one may prefer to consider an equivalent body of concrete, having the same mass. If the concrete weighs 144 lb. per cu. ft., the dimension corresponding to b becomes,

$$b' = 0.38 \sqrt{hy} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

The dotted curve in Fig. 3 indicates this body of concrete. Like the body of ice that was described, it should be imagined as divided into layers resting on one another without friction. The picture then represents no addition to the strength of the dam, but a proper addition to the mass.

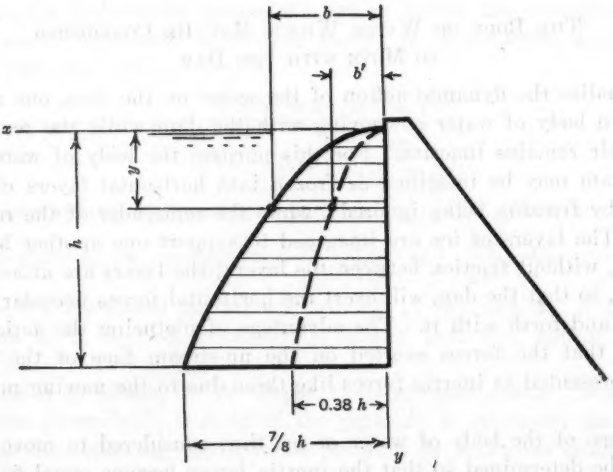

FIG. 3.—BODY OF WATER WHICH MAY BE CONSIDERED TO MOVE WITH DAM. DOTTED CURVE SHOWS EQUIVALENT BODY OF CONCRETE.

ACKNOWLEDGMENTS

This study was made in Denver, Colo., at the United States Bureau of Reclamation in connection with the design of the Hoover Dam, during a leave of absence of the writer from the University of Illinois. All the important designing work of the Bureau of Reclamation is done in Denver, under the general supervision of J. L. Savage, M. Am. Soc. C. E., Chief Designing Engineer; all engineering and construction are under the general charge of R. F. Walter, M. Am. Soc. C. E., Chief Engineer, with headquarters in Denver; and all the activities of the Bureau are under the general charge of Elwood Mead, M. Am. Soc. C. E., Commissioner, with headquarters in Washington, D. C.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

WESTERN HIGHWAY PRACTICE, WITH SPECIAL REFERENCE TO CALIFORNIA[1]

BY C. S. POPE,[2] M. AM. SOC. C. E.

SYNOPSIS

The field of Western highway construction is so widespread and each locality offers so many varied problems that it is only possible to cover a small part of it. The writer, therefore, does not propose to cumber the already voluminous literature of highway engineering with repetitive descriptions of problems or the methods used in the solution of problems already common to highway engineers from their own experience or reading. The paper contains an outline of the methods used in solving certain construction problems of a nature unusual to Eastern terrain and to indicate some of the controls which have been applied in the Western States to those forces of Nature which, unless properly curbed, are destructive to the works of Man.

While the general theme of this paper is devoted to construction designed to insure protection against stream erosion and floods, cloudbursts, and the destructive action of the sea, the writer has also presumed to touch briefly upon the subjects of highway financing and of clearing right of way previous to construction.

FINANCE

Of all problems facing the Western States, that of finance is probably the most serious. The large areas of public lands, the scant population in many locations, the long stretches of country on which no local need of business or political weight urges the expenditure of funds, and the expensive grading and bridge construction which are often necessary for even the most elemental road construction, place upon Western highway engineers a serious

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construction problem. Competition in high ideals in road design and construction has necessitated an orderly procedure in the use of such funds as may become available.

Western highway engineers now realize that no highway system will ever be entirely completed, but that it will be added to, rebuilt, changed, and improved to the advancing needs of traffic farther into the future than any man can predict. Through the gasoline tax a source of steady income, automatically adjusting itself to the finances required, has been set up, thus enabling construction programs projected from 10 to 50 years into the future to be outlined with a moderate assurance that they will be carried forward. Whether a gasoline tax imposing a moderate tariff on motor vehicle owners will furnish sufficient funds to meet the demands of highway users, without the assistance of bond issues, is a question which time will answer.

The application of the budget plan of expenditure, however, and the assertion of the principle of preferential road construction founded on a study of the economic and engineering justification of each road under consideration has done much to clarify the allotments of funds in a rational manner.

In 1930, California began planning its road expenditures on the basis of the needs of the succeeding ten years, in actual definite projects, and the State authorities have been looking ahead very much farther than that in its prospective highway plans.

Clearing the Right of Way

Present-day highway engineering literature devotes itself quite completely to problems such as soil studies, drainage, paving design, and similar subjects relating possibly more to the highway already graded and constructed under average conditions than to the highway in process of construction. Many Western highways present additional interesting construction problems which have possibly not been given so much attention. Among the first of these is the clearing of the right of way previous to construction.

The problem of clearing brush and trees of medium size and thickness of growth is not unusual in many States. Such conditions as those encountered in the Northern coastal highway construction are, however, somewhat unique. Not only does dense undergrowth occur, but trees of very large size, both standing and prone, which must be felled or cleared away. Normal clearing costs where moderate sized trees and scattered underbrush are to be removed, may be stated as about $250 per acre. In the heavily timbered country of the Northern California coast, the clearing may be as high as $1 200 to $3 000 per acre.

Clearing may include grubbing and the removal of all stumps in the case of trees prone to rotting, but in many cases redwood stumps are not removed because of their known long life. Where stumps are removed, the allowance which must be made in excavation computations for filling stump-holes is often a matter of considerable moment.

Trees of moderate size are pulled over with tractors, but in handling trees up to 18 ft. in diameter, other methods are used. The organization for such

work is divided into three groups of men known as "fallers," "buckers," and "blocking" crews. Trees which can not be uprooted with tractors are felled by the "falling" crew and the stumps are afterward blown out or blasted and removed in sections by the "blocking" crew.

Falling is somewhat of a problem when numerous trees more than 200 ft. in height and averaging 12 to 16 ft. in diameter must either be felled in a 100-ft. right of way or must be dropped so that they will not injure trees which are outside the right of way. The work of falling is usually done by two "fallers" to each tree, working with axes and saws. The tree is first heavily notched with axes, after which the trunk is cut through with cross-cut saws. During the sawing, wedges are driven into the saw-cut to prevent the tree from pinching the saw and to insure falling in the proper direction. The final cuts, however, determine the direction of the fall, and it is here that the ultimate skill of the "faller" is exercised.

Following the "fallers," the "bucking" crew trims the felled tree and cuts it into merchantable lengths with drag saws. The stumps are then blasted and logs, stumps, and débris are removed from the right of way by the "blocking" crews.

On one project it was necessary to remove about 1 800 trees larger than 12 in. in diameter and ranging up to 18 ft. in diameter. The cost of clearing approximated $7.00 per 1 000 ft. b. m. of standing timber, while stump removal averaged about $6.50 per stump.

Protection from Elements

Some of the more unusual elements against which it is necessary to protect highways in the Western States are floods, cloudbursts, sea waves, moving sand dunes, and earthquakes.

Floods may be either those caused by the rise of large rivers, or by the sudden flow of water, in otherwise normally dry stream channels, due to cloudbursts.

Structures to furnish protection against cloudbursts consist either of a system of "dips" or depressions in pavements, which permit a temporary overflow across the pavement. By means of these structures the pavements are protected from scouring by means of overflow aprons and cut-off walls; or a system of channels and deflecting dikes which pick up the broad but heavy flow occasioned by cloudbursts and conduct it through control channels over which trestles have been constructed.

Protective structures for sea waves take the form of booms anchored to piles, heavy random rip-rap, hand-placed rip-rap, sheet-piling of wood or concrete, or other bulkheads, groynes of wood or steel, jetties, wave breakers, slope paving, and sea walls.

Protection against moving dunes is generally obtained by a study of their habits and movements. Some may be avoided by a change of location, others by a raise of grade, and others may be removed by wind action or with the use of equipment.

As far as the writer has observed, there is no known cure for earthquakes except good construction which includes the use of a suitable number of expansion joints in concrete paving, the clearing of all hillsides of hanging rock, and solid construction of all fills in earthquake country.

Flood Protection.—The character of structure which may be used with success in flood-protection work is determined partly by the volume of water to be handled and partly by the character of the soil, sand, gravel, or other débris transported by the flood.

The more common of the most destructive floods in the West occur in arid and sandy regions and are usually accompanied by the transportation of considerable quantities of sand. In many cases, this prevents solid structures from being used successfully because of lack of solid foundation, or it makes them useless for the reason that stream beds are filled up and changed. There has been considerable use, therefore, of structures of a type which are flexible or which may be easily replaced.

Brush and Wire Fencing.—Among these is the brush and wire fence constructed of two rows of galvanized-iron pipe driven deep into the sand parallel with the stream. This type is shown in Fig. 1, which is designed for bank protection in slow-flowing streams through sand and gravel. Fencing of wire mesh is fixed to the pipe to retain the brush which is placed between the rows of fencing. The brush is weighted down with stones to insure its settling and to fill any washouts which might occur under the fencing.

The California Division of Highways has developed a very definite system of installing this type of fencing. Galvanized-iron pipes 2 in. in diameter are spaced 5 ft. apart in line and strongly braced with 2-in. pipe both laterally and in line with the fencing which consists of heavy wire mesh. The parallel fencing is usually placed at a distance of $2\frac{1}{2}$ to 5 ft., and the intervening space is filled with alternate layers of willow brush and stone. A layer of brush, $2\frac{1}{2}$ ft. thick, will compact to a layer, 1 ft. thick, when tamped and loaded with sufficient stone to hold it down.

A variation of the willow and stone filler is the use of stone-filled "sausages" of wire mesh, which are laid on top of each other between rows of fencing. Naturally any break in the wire mesh "sausage" casing, however, brings about destruction of the "sausage."

Another common practice is the construction of mats of wire mesh and cobbles. The slope is first dressed to an even surface on a slope of about 1 on 2, and on it is laid a section of wire mesh with tie-wires at suitable intervals. A coating of cobbles, 4 in. or 8 in. thick, is then laid on the mesh and a second mesh panel is wired down with the tie-wires on top of the cobbles. The mats must be fixed to the top of the slope to prevent their sliding, and this is often done by driving pipes into the slope at 6-ft. intervals and wiring the mats in place. It is also necessary that the mats extend over the stream bed sufficiently to blanket any possible areas of scour. This construction gives a flexible mat which, to a certain extent, will follow any undulations which may occur due to washouts. It has been extensively used in storm-protection work in Los Angeles County, California, but not

FIG. 1.—PIPE, WIRE MESH, AND BRUSH FENCING FOR BANK PROTECTION.

FIG. 2.—WIRE MESH MATS FILLED WITH GRAVEL FOR BANK PROTECTION.

FIG. 1.—PIPE WORK BLOCK AND BRONZE PACKING FOR BLAKE PUMPS.

FIG. 2.—WIRE MESH PLATE-FILTER MADE CHEAPLY FOR BLAKE CONDENSERS.

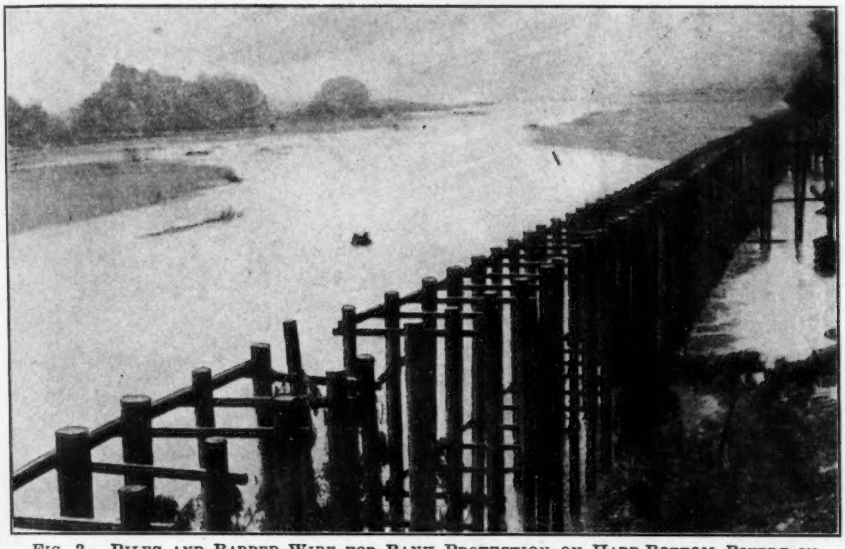


FIG. 3.—PILES AND BARBED WIRE FOR BANK PROTECTION ON HARD-BOTTOM RIVERS IN TIMBERED COUNTRY.

FIG. 4.—TETRAHEDRONS FABRICATED FROM STEEL RAILS FOR SHORE PROTECTION.

FIG. 5.—CONCRETE MUSHROOM ANCHORS TO HOLD RAIL TETRAHEDRONS IN PLACE.

FIG. 6.—REINFORCED CONCRETE TETRAHEDRONS FOR BANK PROTECTION IN A STREAM WITH A WINDING CHANNEL.

to any great extent by the highway organization. The cost is stated as about 25 cents per sq. ft. Fig. 2 is a view of a section of wire-mesh mats filled with gravel to protect banks of gravel and sandy soil in Southern California. The type is effective to withstand intermittent floods that carry boulders in their paths.

Reinforced Concrete Slope Paving.—Concrete slope paving is extensively used for bank protection on California highways. It is usual to place a suitable toe-wall of sufficient depth to guarantee the paving against scour. The considerations governing the thickness and reinforcing details of this paving are the slope height and the character of the detritus carried by the stream. In some localities it has been found desirable to make the thickness of walls more than 6 in. in order to resist the action of large boulders brought down by floods, but in most cases a thickness of 6 in. is sufficient.

The cost of reinforced concrete slope paving varies with the local conditions, but experience in California shows it to average 25 to 30 cents per sq. ft. for 6-in. thickness and 20 to 25 cents per sq. ft. for 4-in. thickness, without including toe-walls.

Jetties.—Deflecting jetties constructed of piles with wire mesh, or barbed wire nailed to them, or of stone-filled planking, were at one time considered a satisfactory type of stream protection. At present, however, their use is confined largely to the northern part of California where the character of the stream bed is more stable than that in the southern part and where the possibility of scour under the fencing is remote. When properly constructed, jetties prevent bank erosion and are of considerable value. An example of this construction is shown in Fig. 3.

Tetrahedrons.—Where the character of the detritus carried by the stream is rather light, such as the Colorado River silt, or fine sand, the use of skeleton tetrahedrons of steel or concrete has been found very effective for stream deflection and bank protection. These tetrahedrons are placed in line across a point that is giving trouble due to scour and are usually anchored with cables to insure their being maintained in position. Their function is to interrupt the flow of the stream in such a manner as to cause silt to deposit in the eddies back of them, and they have performed very successfully in a great many locations. Those used on the Colorado River by private interests were constructed from steel rails, 30 ft. in length, and were used successfully in deflecting this large stream where other means had not been successful (see Fig. 4). They were held in line by the use of steel cables attached to mushroom anchors of concrete (see Fig. 5). The method of placing had a very definite influence on their successful use. This consisted of placing the obstructions one after another down stream, which caused the silting process to begin at once and prevented further scour.

On highway work, a row of reinforced concrete tetrahedrons which had been placed to deflect the stream on the Ventura River was the means of saving a considerable section of highway during the St. Francis Dam disaster (see Fig. 6).

Stone-Filled Steel Wire Baskets.—In one location, a mountain river in heavily timbered country, metal baskets, 7 ft. square, and of varying depths,

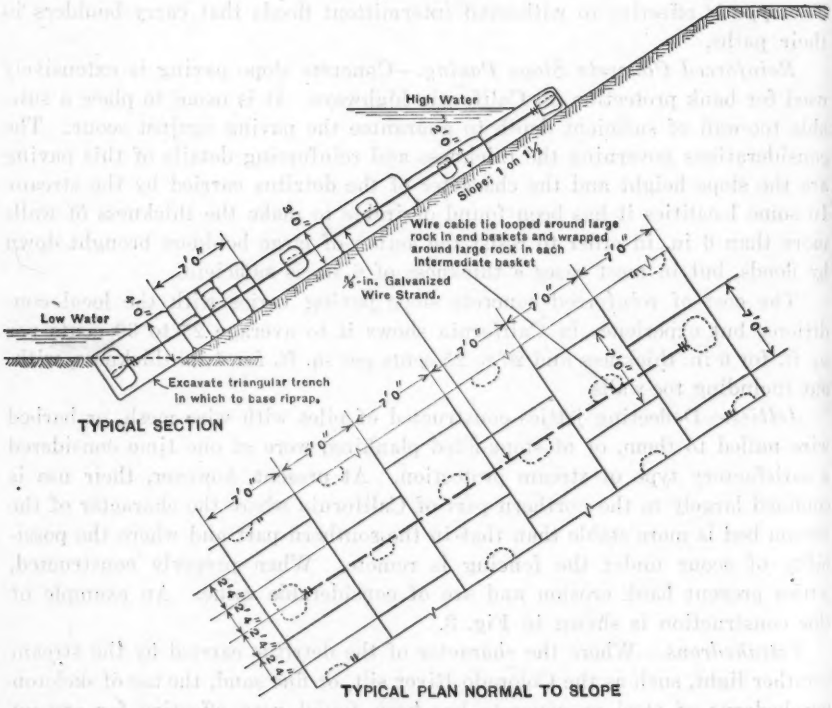

High Water
2'-0"
Slope: 1 on 1½
3'-0"
Wire cable tie looped around large rock in end baskets and wrapped around large rock in each intermediate basket
7'-0"
½-in. Galvanized Wire Strand.
Low Water
2'-0"
Excavate triangular trench in which to base riprap.
TYPICAL SECTION
TYPICAL PLAN NORMAL TO SLOPE

FIG. 7.—BASKET RIP-RAP AND WIRE CABLE TIES.

filled with stone, were installed under plans made by the U. S. Bureau of Public Roads. A typical plan and elevation are given in Fig. 7 and approximate quantities are listed in Table 1.

TABLE 1.—APPROXIMATE QUANTITIES IN RIP-RAP BASKETS

Depth of baskets, in feet	4	3	2
Rip-rap capacity, in cubic yards	7.26	5.45	3.63
Weight of wire mesh, in pounds	150	138	126
Weight of wire ties, in pounds	3.6	3.0	2.4

Slope paving of hand-placed stone with appropriate toe-walls has been extensively constructed in a number of suitable locations. On many streams the use of random rip-rap of considerable weight has been advantageous as a method of protection. Fig. 8 shows two types of slope protection suited to canyons that are subject to boulder-carrying floods.

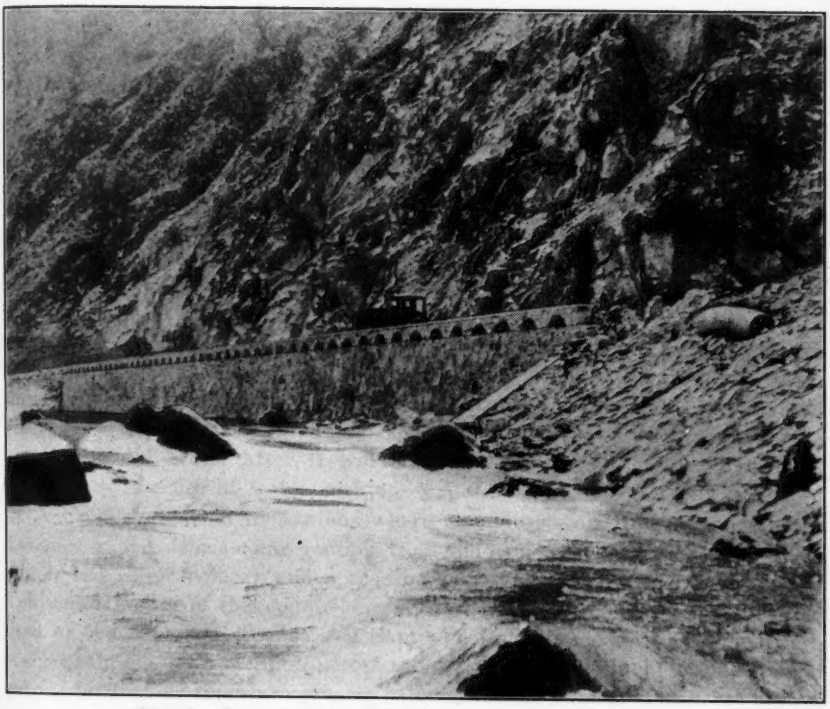
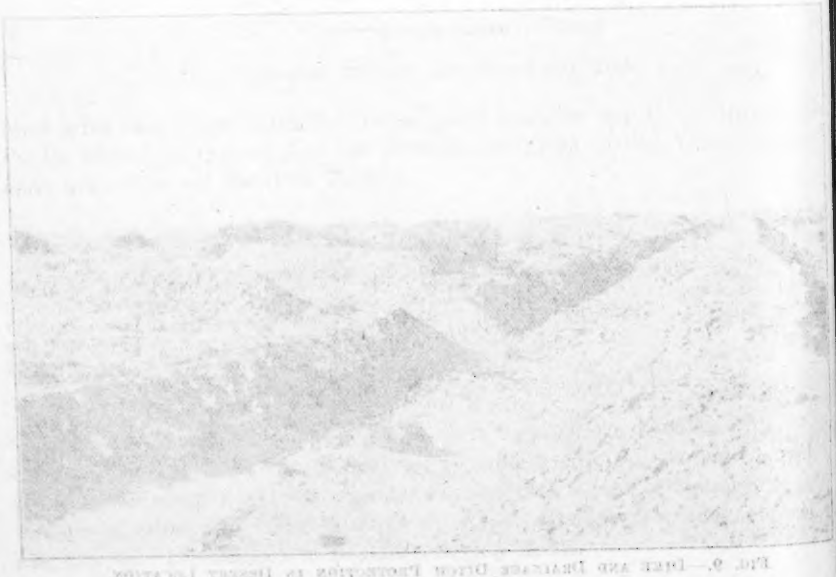


FIG. 8.—RUBBLE WALL AND HAND-LAID DRY SLOPE PAVING.

FIG. 9.—DIKE AND DRAINAGE DITCH PROTECTION IN DESERT LOCATION.



Cloudbursts

One of the most unusual problems in California is the protection required against cloudbursts. It is the opinion of many engineers that these occur in approximately the same areas within a reasonable range of time. Since in many localities the hills and mountains are entirely denuded of any verdure which would retard the flow of water, the volumes of water to be handled are very large and the run-off is very rapid. The water often appears without previous intimation that a cloudburst has occurred. The first knowledge which the observer has of the approaching danger is the appearance of a wall of water and mud sweeping down the canyon. The method pursued in the past was to construct paved "dips" across all locations where the profile indicated that storm or cloudburst run-offs were usual. This, however, proved unsatisfactory in many cases because of extreme scour which occurred at the overflow aprons, and also the great expense of cleaning débris from the dips and roadway after each storm. It seemed practically impossible to check the velocity of the water after it had crossed the pavement, either by cut-off walls or by the use of water cushions.

A usual formation in sections where cloudbursts are frequent shows the presence of flat débris cones issuing from the canyons (or other sources from which the water comes) and spreading out fan-wise into the lower lands. Taking advantage of this condition, the State has adopted the system, formerly used by the Atchison, Topeka and Santa Fé Railway Company, of constructing pick-up channels or dikes across the drainage lines of the débris cones in such a manner as to secure greater velocity in the channels than is afforded by the general slope of the cones (see Fig. 9). The channels, of course, must be located close enough to the highway to prevent any considerable quantities of water falling between the intercepting system and the highway. The results thus far obtained have been extremely satisfactory in restraining all the water from crossing the highways, except at certain designated points, known as controls, where wooden trestles are constructed along the highway for the passage of streams. The system of drainage thus developed has given rise to a nomenclature for its various parts. The source of the water is usually designated as the "cone," the main drainage channels as "ditches," feeder channels as "sub-ditches," and the outlets at which water is concentrated and carried across the highway as "controls."

In Fig. 10 the highway location is parallel to a desert valley. The main drainage is through a series of dry lakes or sinks. Tributary drainage is shown emerging from the mountains to be intercepted by one or two main channels that break up into interlacing overflow channels. The storm-protection ditches are located so as to obtain the maximum fall. Associated with the channels is often a system of dikes for directing the flow. In other places where large open valleys are encountered in desert locations, and storm flow occurs as broad sheets of water, it has been necessary to construct extensive dike systems and to conduct the water under the highway by means of wooden bridges or trestle structures, as before stated.

A desert valley of this type has natural drainage coming from the mountains on each side of the valley and discharges usually into a series of lakes and sinks. These sinks or dry lakes are formed in most desert valleys by the débris deposits from major side-tributary drainage which has cut off and backed up the main valley drainage.

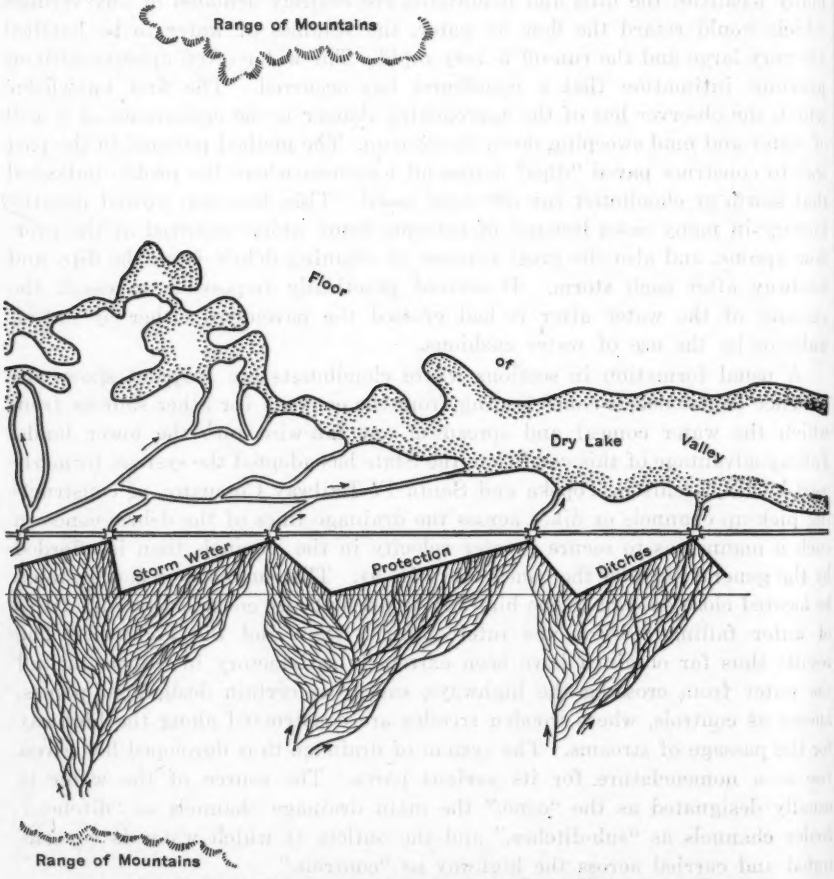


FIG. 10.—TYPICAL HIGHWAY LOCATION, WITH STORM-PROTECTION DITCHES.

The cost of ditching for the type of work done on débris cones is about $1 500 per mile. The cost of the wider ditches with dikes, such as are used in the wide desert valleys, is about $2 500 per mile. The cost of timber bridges and trestles of 20 to 24-ft. creosoted piles and of redwood superstructure of 24-ft. roadway, varies from $100 to $125 per ft. of length. Highway protection by this system costs from $3 000 to $3 800 per mile.

SEA PROTECTION

The sea has been a constant menace to construction in many localities, but it is quite common for highways to be located where this danger can be averted. However, in a number of places considerable lengths of highway

have been built along the sea, and it has been necessary to provide protective structures. "Booms" have been chained to piling in some locations, but this at best is a temporary expedient and is not now used in highway work.

Random rip-rap with most of the stones weighing not less than 5 tons has been used to some extent on highway work where the action of the waves and tides is normal to the coast line; but the type of equipment available on highway work makes the handling of heavy stones extremely difficult. This situation arose in connection with certain work in Southern California along the rocky coast north of Santa Monica where protection of slopes against sea action became necessary. It was decided that, since large rock suitable for both rip-rap and blanketing the slopes could not be handled at the site of the work, a different plan of protection should be used. Accordingly, a design was prepared by which concrete cells should be constructed and lowered into place and afterward filled with concrete to increase their weight. The reinforced concrete cells finally decided upon were 10 ft. long, 5 ft. wide, and 3 ft. high, and their estimated weight about 3½ tons before filling, or 12 tons in place after filling. Their cost in place was about $95 each, or, approximately, $8 per ton.

After the cells had been set in place and before they were filled with concrete it was necessary to grout the boulders on which they were set with quick-setting concrete in which cement and hard wall plaster were combined in equal parts in order to insure sealing of the concrete against wash from below. When a cell had been filled completely (see Fig. 11), it was again necessary to protect the fresh concrete against wave wash by boarding the top.

The shore line at this location is entirely exposed to the full sweep of the Pacific Ocean and consisted of rocky bluffs with a steep slope into deep water. The design in this case called for the construction of a toe-wall below high tide, the paving of the slope in front of the toe-wall to below low water, and the construction of reinforced concrete slope paving, 9 in. thick, based on the toe-wall and ending in a parapet designed to throw back running waves. It was possible to place the concrete cells along the tide line, but owing to the heavy run of the sea, it would have been impossible to have built the toe-wall or the slope walls without some protection. Accordingly, such leveling and removal of boulders as was possible was done at low tide and the concrete cells were then sunk into line on the roughly prepared foundation and were filled with concrete and heavy stone between tides.

When the cells had been poured and the beach boulders grouted, enough protection was afforded so that sufficient time was allowed in spite of the rising tide, to pour the base of the toe-wall to a height of 1½ to 2 ft., leaving from 1½ to 2½ ft. to construct on top of the base, the cells acting as a form on the outside. After the section of base was constructed, the top section was poured into the forms built on the base poured previously. These forms were built while the other work was proceeding. The toe-wall was poured with Class C concrete, 4.2 sacks per cu. yd. Steel bars, ½ in. by 4 ft., were used in the toe-wall. These bars were embedded in the base at 12-in. centers extending up through and out of the top of the wall to tie the toe-wall to the slope paving.

To give a better understanding of the aforementioned procedure, it is necessary to call attention to the great difficulty created by the tides. This work had to be carried on during the winter months, the most unfavorable time of the year, in order not to delay the slope paving. All the work was at a low elevation on the beach, varying from +3 to zero, and occasionally below zero. The low-tide elevation varied from approximately + 2 to − 1.5, with an average of zero. Favorable low-tide conditions were often nullified by high winds and a consequent heavy surf.

A maximum of work had to be done at low-tide periods. Excavation was usually begun 2 to 3 hours before low water, while the waves were still breaking over the location of the wall. All the men were soaked from head to foot. Occasionally, a man would be slightly injured from being upset by a wave. The work could be pursued until the tide got too high, 2 to 3 hours after low tide, giving from 4 to 6 hours per day in which effective work could be done. It was found, in general, that excavation or form work could not be left over night, as the excavation would be washed full of sand, and the forms washed away or badly damaged.

Broken shifts were often necessary on account of tide conditions. Night work was made possible by the use of carbide lights. Moving the derrick and equipment, hauling cement and other materials, building concrete chutes, etc., was done at high tide, when no work could be done on the beach. The difficulties encountered in this work under such conditions cannot be overemphasized.

Behind the protection of the concrete cells it was possible to excavate for the construction of a toe-wall on which the reinforced concrete slope wall could be placed. This slope wall was made 9 in. thick; it was heavily reinforced and was capped by a wave-deflecting parapet designed to throw the waves back on themselves (see Fig. 12). Previous to placing the slope paving, a bedding of stone was laid roughly on the earth slopes. The work could only be done during periods of calm weather and, on this account, was handled by day labor at a cost of about $24 per ft. of wall, including excavation, cells in place, toe-wall, slope paving, and parapet. While the 9-in. slope paving had survived a number of heavy storms up to August, 1931, there has been considerable erosion due to the action of water-carried gravel and sand on the slabs, and the protective work cannot be said to be an entire success.

The use of sheet-piling of either wood, steel, or concrete, on State work in California has never met with much success, and it has not been used extensively in sea protection.

In locations where the run of the tide is parallel to the shore line and where sandy beaches alternately form and depart during storms, groynes have been used extensively along the south coast and in many locations the beach has been restored by their use. Fig. 13 shows a beach immediately after groynes were constructed and Fig. 14 shows their effect some time afterward. The State construction usually contemplates the use of wooden structures, but some of the private beach owners have used steel sheet-piling with considerable success. The usual State construction contemplates the

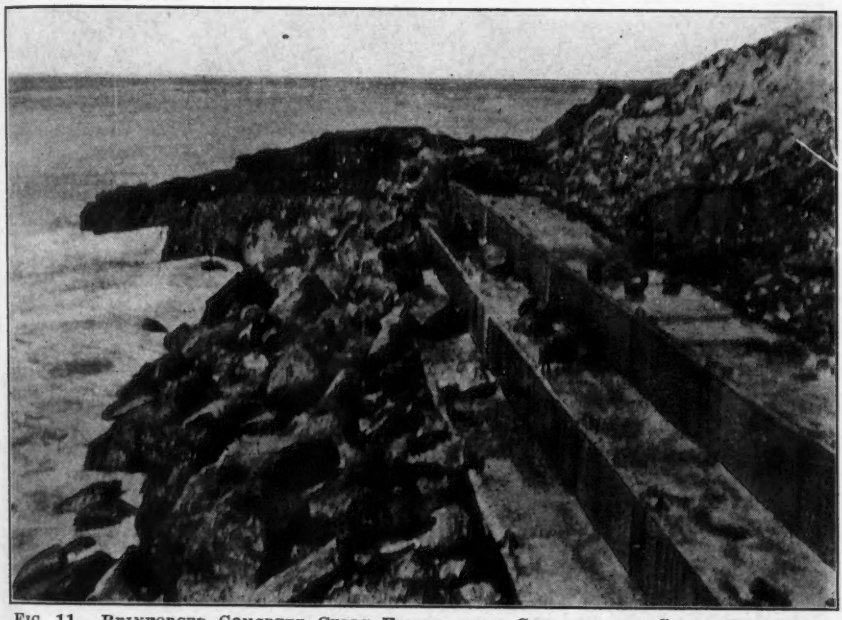


FIG. 11.—REINFORCED CONCRETE CELLS FILLED WITH CONCRETE FOR SHORE PROTECTION.

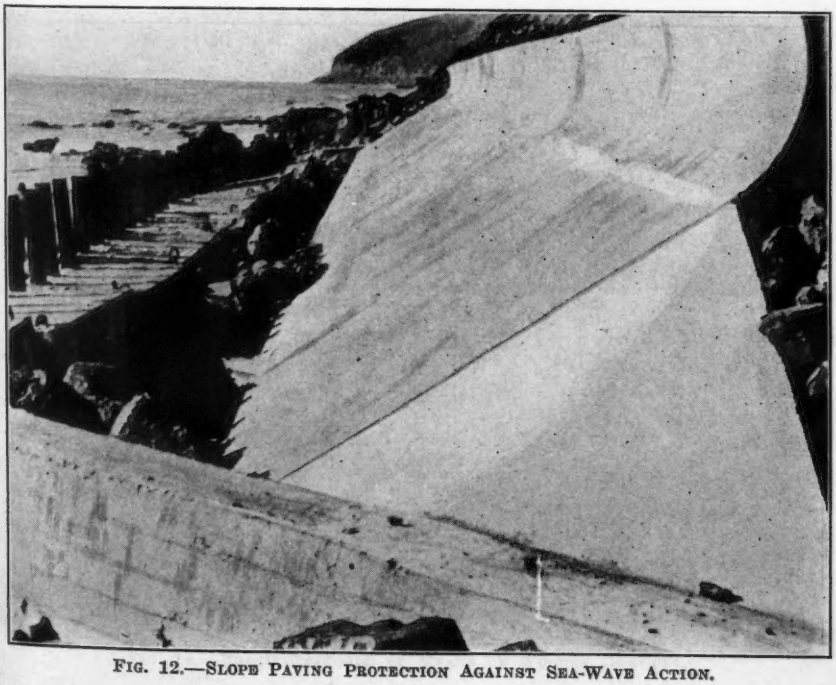
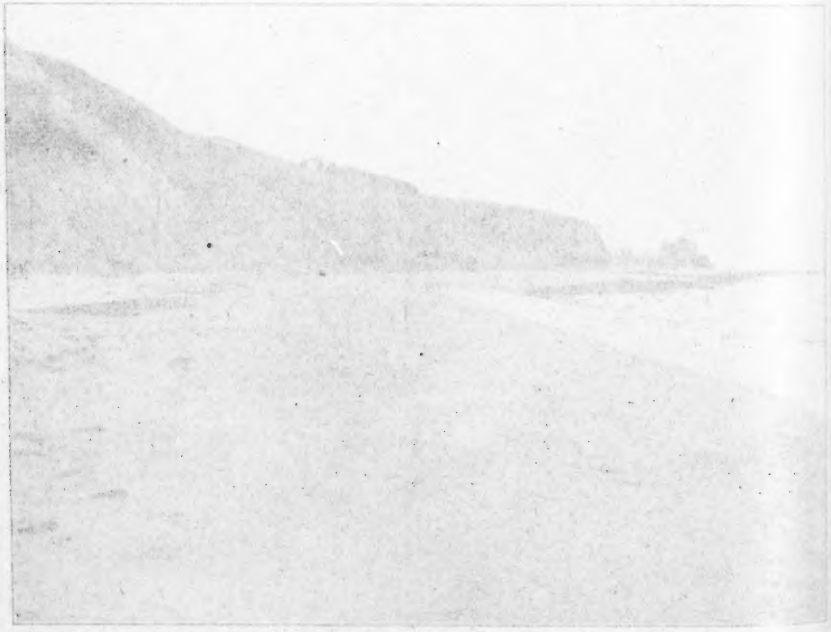


FIG. 12.—SLOPE PAVING PROTECTION AGAINST SEA-WAVE ACTION.

FIG. 13.—WOODEN-GROYNES PLACED TO RESTORE A BEACH.

FIG. 14.—BEACH RESTORED BY MEANS OF WOODEN GROYNES.



driving of posts at suitable intervals in the beach, at right angles to the coast line, to support planking. Planking is added as the beach builds up until the maximum height desired is obtained. The spacing of groyne lines varies, but may be said to be from 50 to 200 ft., depending on the direction of the coastal current.

Sea-Walls.—In one or two locations, the State has constructed heavy sea-walls of monolithic concrete of sufficient strength to withstand the action of the heaviest waves. These have been built with the usual throw-back design of top which has proved effective wherever tried.

Sand Dunes

In a number of places the California Division of Highways has been faced with the problem of control or elimination of sand dunes which have formed not only a serious burden on maintenance, but have been extremely dangerous to traffic. It is quite usual for a sand storm to place a dune, within a few hours, across the highway, several feet in depth, which would require constant labor to remove. On one particular dune the State had an annual expense of about $7 000 for keeping the road clear over a section of a few hundred feet.

A study of the habits of dunes and their movements, made over a number of years, demonstrated that they could be conquered if proper means were used. In many given localities most of the dunes were found to move in a certain direction and then attain a height which was seldom exceeded elsewhere. By the construction of a high-grade line over the worst section of dunes all dangerous drifting was entirely eliminated and the road was clear at all times thereafter. In another location, a study of the dune indicated that it could be removed by wind action by cutting channels at suitable locations. This work was done and the dune was removed at a cost of a few hundred dollars; the removal of similar dunes has since been accomplished by the same method at a very low expense to the State.

Earthquakes

The occurrence of earthquakes offers an unusual problem to the highway engineer. Some engineers in California are of the belief that a number of concrete highways have been shattered by these disturbances and that the construction of proper expansion joints will eliminate this danger. In earthquake country, highway engineers have found it necessary to construct fills with the greatest solidity and also to clean up slopes so that there are no hanging rocks in any location. On one job in San Diego County, the State suffered a loss, due to combined earthquake and cloudburst, of more than $30 000 in one storm which brought down enormous boulders on a section of highway about eight miles in length. During the Santa Barbara earthquake, a concrete pavement was shaken from side to side until there was a gap about 4 to 6 in. wide along each side of the highway in the vicinity of Santa Barbara. The highway had also buckled at a number of places and had settled at bridge abutments.

Generally speaking, however, the danger due to earthquakes is not serious so far as highways are concerned.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

LOW-COST BITUMINOUS ROADS [1]

BY WALTER N. FRICKSTAD,[2] M. AM. SOC. C. E.

SYNOPSIS

In the development of modern highway systems there is a serious need for a low-cost secondary road that will accommodate a limited heavy traffic and an unlimited light traffic. In the western part of the United States a light type of asphaltic binder known as fuel oil has been successfully used to treat crushed aggregate. Whether mixed at a central pug-mill plant, or on the road itself, the costs are about the same, and the result has proved economical in first cost and maintenance charges.

After summarizing the status of this field up to October, 1928, the writer discusses the advances from that time to date (1931), describing construction methods, specifications for materials, and necessary equipment.

INTRODUCTION

Two types of highway work, involving treatment of fine crushed aggregate with a light grade of asphaltic binder known as fuel oil, may be classified as surface treatment and oil-mixing treatment, respectively. In an unpublished paper,[3] entitled "Bituminous Treatment of Fine Crushed Rock and Gravel Roads," Thomas E. Stanton, Jr., M. Am. Soc. C. E., has described these treatments and the extent of their application in the Western United States up to October, 1928. Mr. Stanton's paper is summarized briefly herein.

Surface Treatment.—This is sometimes called penetrative treatment because the binder is absorbed into the upper strata of aggregate. The first step in construction is to prepare the base and this implies repairs, thorough compaction, and thorough cleaning. Approximately, $\frac{1}{4}$ or $\frac{3}{10}$ gal. of asphaltic fuel oil per sq. yd. is spread on this prepared base. The oil should contain 60 to 70% of 80-penetration asphalt having a specific viscosity (Engler) at 50° cent. of 10 to 25. The oil is allowed to penetrate the upper strata of the

[1] Presented at the meeting of the Highway Division, Sacramento, Calif., April 24, 1930. Written discussion on this paper will be closed in **February, 1932**, *Proceedings*.

[2] City Engr., Oakland, Calif.

[3] Presented at the meeting of the Highway Division, San Diego, Calif., October 4, 1928.

metal; sometimes stone chips are spread to protect traffic. After a suitable interval another application of asphaltic oil is spread. This may be of the same nature as the primer, or it may be of slightly higher viscosity; it may be much heavier, approaching pure asphalt.

The next step is to spread stone chips in sufficient quantity to absorb the oil and then the road is rolled. Details vary considerably, particularly in the quantity of oil and stone chips added, which is greater for the heavier oil. Sometimes the second application of oil is kept to a minimum, usually $\frac{1}{5}$ to $\frac{1}{4}$ gal. per sq. yd.; the stone chips are relatively large, as, for example, $\frac{5}{8}$ to $\frac{1}{8}$ in., and a third spread of oil of approximately the same quantity is applied, and followed by still finer stone chips. Heavy oil, applied hot or emulsified, is considered more durable than fuel oil, particularly against weather damage.

Oil-Mixing Treatment.—In this process the asphalt oil binder is thoroughly mixed with the upper 2, 3, or even 4 in. of surfacing material. The first construction operation is to provide the desired quantity of loose mineral aggregate on top of a firmly compacted base. All the aggregate should pass a 1-in. screen and one-third to one-half of it should pass a 10-mesh sieve. It may be obtained by scarifying if a sufficient supply is already on the road, or it may be added for treatment. The oil is usually spread in three applications, aggregating 1.25 to 2 gal., or more, per sq. yd. After each application the materials are mixed by means of a disk or spring-tooth harrow, or by both. After all the oil is applied, the mixing operation is completed by casting the material into a windrow with a blade grader and moving the windrow back and forth over the road, turning the mass at each move. The oil commonly used contains from 60 to 70% of 80-penetration asphalt. The viscosity is variously specified, generally 25 to 45 (Engler specific) at 50° cent., or 45 to 80.

The quantity of oil varies according to the grading and other characteristics of the aggregate. As in the case of asphaltic concrete the higher proportion of oil is required for the finer material. C. L. McKesson, M. Am. Soc. C. E. has suggested[4] the following equation for estimating the quantity of oil required:

$$P = 0.015a + 0.03b + 0.17c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which, P = percentage of oil required; a = percentage of aggregate retained on the 10-mesh sieve; b = percentage passing the 10-mesh and retained on the 200-mesh sieve; and c = percentage passing the 200-mesh sieve. The quantity of oil is sometimes estimated by Equation (1) and sometimes by appearances, but quite commonly by a modification of the Stain Test so long used in sheet-asphalt practice. After the mixing operation is completed the material is carefully spread with a blade grader to uniform thickness, and is compacted by traffic. During the compacting period, ruts and other irregularities are eliminated by a drag or blade grader until traffic makes no impression and until the road is as smooth as the most carefully constructed of the higher types of pavement.

[4] "Light Asphaltic Oil Road Surfaces," by C. L. McKesson and W. N. Frickstad, *Public Roads*, September, 1927, p. 150.

In addition to the foregoing, a method of plant-mixing, first introduced in California in 1928, should be mentioned, in which the aggregate and the oil are mixed at the source in the ordinary type of pug-mill. The aggregate is not heated, but the oil is warmed enough to facilitate pumping. When the mixed material is placed on the road it is spread by a blade grader and compacted in the same manner as the road mixture. It has been found that the cost of pre-mixing when the surfacing material must be brought to the road is little, if any, greater than road mixing under the same conditions. To those acquainted with the price of asphaltic concrete the low cost of fuel-oil mixture may seem surprising, but the explanation lies in the elimination of heating operations at the plant and the simplicity of handling cold material on the road.

At the end of the 1928 working season approximately 3 600 miles of State highways in Washington, Oregon, California, Nevada, Arizona, New Mexico, Colorado, Utah, Idaho, Montana, and Wyoming had been oil-treated. Of this mileage, 2 330 was surface treated, 1 170 was mixed, and 100 was of miscellaneous types. Some of that classified as miscellaneous was in the nature of penetration macadam. During 1929, 730 miles were surface treated, 1 640 miles were mixed, and 130 miles had miscellaneous treatments. Hence, about 2 500 miles were treated in 1929, making the grand total up to that time, 6 100 miles. Statistics for 1930 and 1931 are not available, but at least 3 000 miles were added each year, giving a total at the end of this (1931) season of more than 12 000 miles. The foregoing figures do not include county roads and small town streets, which aggregate a considerable but unknown mileage.

Development in Surface Treatment

The trend of surface treatment practice is now (1931) very definitely toward the use of heavy oils, with an asphalt content of 90%, or more. Fuel oil is used principally as a primer or as a dust palliative. A few of the less traveled roads are being surface treated with fuel oil alone, but generally this application is followed with heavy oil and coarse stone chips. Thus, there is a reversion to the methods by which macadam roads have been treated successfully since about 1910. Although the process has been satisfactory on macadam, it has not heretofore been commonly successful on gravel or fine-crushed rock. The introduction of a primer seems to have overcome the difficulty.

Some authorities have a strong preference for tar rather than oil, reporting better penetration into the road and more careful curing with less annoyance to traffic. The Highway Department of Oregon has used a light tar for this purpose and reports marked success.

Details of the surface-treatment process are far from being standardized. One of the California State Highway methods is among the most elaborate and may be outlined as follows:

(1) Clean, repair, and compact the base thoroughly;
(2) Spread ½ gal. per sq. yd. of 50 to 60 asphaltic fuel oil. (Specific viscosity (Engler), 10 to 25 at 50° cent.);

(3) Allow the first application to penetrate and add a small quantity of fine stone chips if necessary to protect the traffic and to absorb oil that may settle in depressions;
(4) Spread ½ gal. per sq. yd. of 94 + asphaltic oil at a temperature of approximately 350° Fahr.;
(5) Spread sufficient ½ to ¾-in. stone chips to cover the oil. (120 tons per mile of 18-ft. road is the estimated quantity, but this includes considerable waste due to the effort to save labor);
(6) Roll;
(7) Spread ½ gal. of 94 + asphaltic oil as before;
(8) Cover with ⅜ to ½-in. stone chips. (80 tons per mile is estimated, which includes waste as before); and,
(9) Roll.

The Oregon Highway Department has a similar specification, using slightly smaller quantities of oil. This Department also has a modified method, omitting the third application of oil, but spreading the two sizes of stone chips separately. The Oregon estimate for stone chips is about two-thirds that of California.

The surface-treatment process has made some headway in Washington and in Idaho, in addition to California and Oregon, but the other Western States have not generally adopted this type. The State of Washington, like California, has treated many roads as a temporary expedient, realizing that elimination of dust and conservation of road metal will compensate for a considerable proportion of inevitable failure.

Plant Mixing

Plant-mixed oil and aggregate presents one of the most interesting possibilities among the developments concerning the oil mixtures. It has spread in an experimental way to several other States, and California is firmly committed to this type of treatment for new construction. When the surfacing material had to be brought to the road, the cost of the earliest project was little, if any, more than road mixing. Plant mixing, of course, permits more complete control than road mixing. Some find that the plant mixture lacks the ideal riding qualities of the road mixture, but if this is true the fault can be corrected by a moderate amount of manipulation while spreading.

To be economical, the mixing must be performed without any extra handling of the material except its passage through the mixing box. Elimination of the need for heating the aggregate and the consequent opportunity for spreading and compacting, without consideration of loss of heat, keeps the mixing cost surprisingly low. It is reported, however, that the power required to operate the mixer is about one-half more than that required for the same quantity of heated aggregate and asphalt.

Equipment for Field Mixing

The machinery used for road mixing continues to vary in type. Most of the States are equipped with tractors and blades for snow-fighting and ordinary maintenance, and have simply transferred this equipment to oiling operations during the appropriate season. Use for these other purposes has

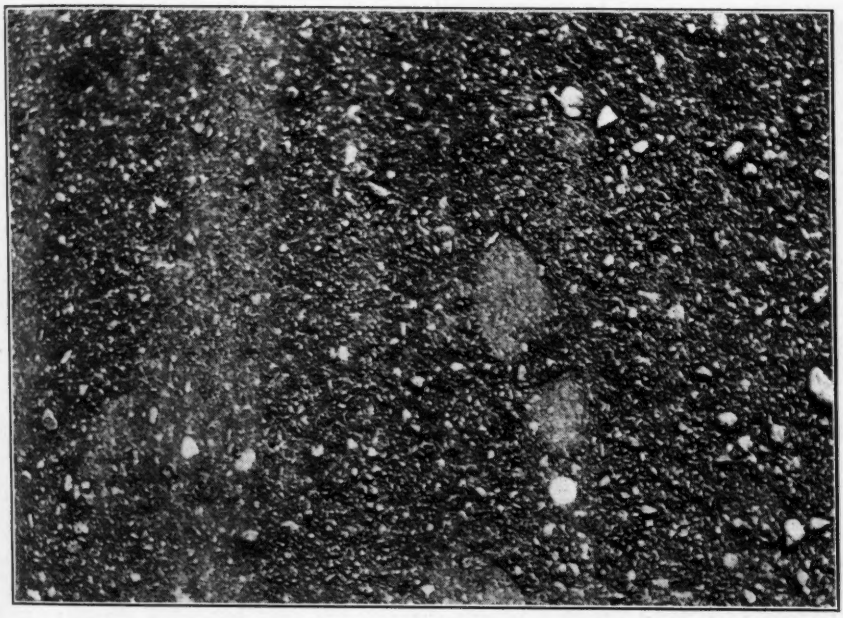


FIG. 1.—TEXTURE OF FIELD MIXTURE AFTER SPREADING.

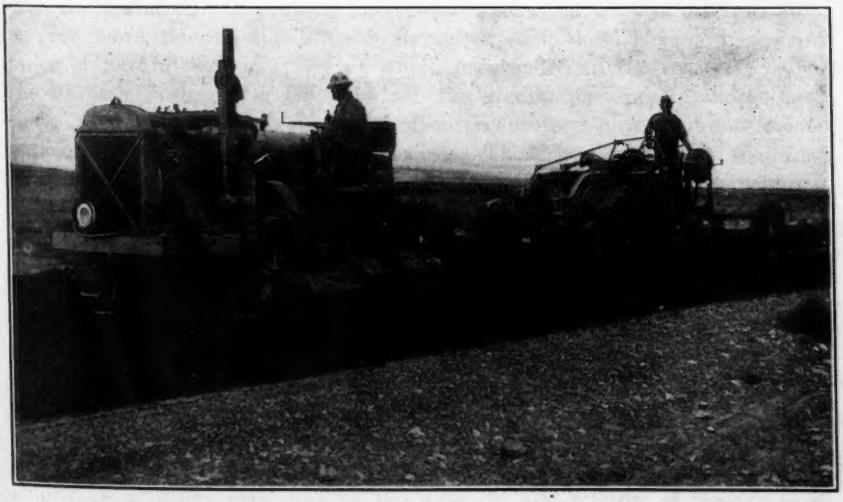


FIG. 2.—FIELD MIXING, MOVING MATERIAL FROM ONE WINDROW TO ANOTHER.

influenced the purchase of equipment for oil mixing. One State, however, has equipped itself primarily for oil mixing, and has adopted the tractor or dual-drive type of self-propelled blade. A group of four of these motor graders is extremely efficient. The best work is done with blades 10 or 12 ft. long.

Regardless of the type of machinery, workmanship has steadily improved. The first thought was to regard mixing as a rather haphazard process, and too often it was so in fact. Since the beginning of 1929 neatness and system have been notable. Partly oiled material is picked cleanly from the base and turned over and over in accurately lined windrows; during the final turnings, a small quantity is allowed to remain on the base, and the final mixing takes place without the incorporation of uncoated particles. It is then spread quite accurately to uniform thickness (see Fig. 1). Even where the grading of the aggregate varies from station to station, with a consequent necessity of adjusting the quantity of aggregate drawn into the mixture, the windrows become remarkably uniform in color before spreading when properly manipulated. Observing the work of a skillful crew, road mixing can no longer be regarded as haphazard. One step in this process is shown in Fig. 2, which illustrates work on a Nevada project.

Grading of Aggregate

The grading of aggregate has not tended to become standardized, which confirms the opinion that a wide range is acceptable for oil mixing. First experiments indicated that the mixture should conform to that of asphalt concrete. Since the maximum size was fixed at ¾ in., or 1 in., it followed that the proportion of material passing a 10-mesh sieve would approximate 40 to 50%, of which perhaps one-fifth would pass the 200-mesh sieve. However, successful oil-mix projects have been constructed with as much as 80%, or even more, passing the 10-mesh sieve. In fact, there is record of many miles of natural soil mixtures of which practically all the material passed the 10-mesh sieve, and a few in which the greater part passed the 80-mesh sieve. On the other hand, successful oil-mixed projects have been constructed with only 25% passing the 10-mesh sieve. There is good reason, therefore, to believe that gradings which correspond not only to asphaltic concrete, but to Topeka mixture and sheet asphalt, may be successfully mixed with fuel oil.

The proportion passing the 200-mesh sieve is an important element. This has been as low as 3 or 4%, and higher than 20 per cent. Apparently, the 200-mesh material should fill the spaces between the larger particles sufficiently to reduce the size of voids to the minimum and increase their number to the maximum. Studies of the fundamental physical characteristics of asphaltic concrete and sheet-asphalt mixtures have been made and the results have also "shed light" on the subject of oil mixtures.

In particular, it is hoped that further research pertaining to the characteristics of fillers may disclose the quantity and quality of the 200-mesh material desirable for oil as well as asphalt mixtures.

Quality and Proportion of Oil

Success is being had with asphaltic oils from the Rocky Mountain area, although their characteristics are somewhat different from the California product. So many factors affect the success or failure of mixtures that the influence of the oil is difficult to segregate. Plant mixtures offer a better opportunity to study the effect of oil than road mixtures. Undoubtedly, oils differ in their suitability, and a strong effort should be undertaken to ascertain the reason.

After the first season of oil mixing the medium grade of fuel oil with an Engler specific viscosity of 25 to 45 at 50° Fahr., carrying 60 to 70% of 80-penetration asphalt, was generally recommended. Subsequently, there has been a marked tendency to raise the viscosity to 55 to 85, which is the grade generally used for plant mixing, and the asphalt content to 65 per cent. The higher viscosity generally carries with it higher specific gravity, and perhaps higher ductility, but adds somewhat to the difficulty of mixing, particularly in cool weather.

The quantity of oil which should be used is much debated for the reason that aggregates vary widely in grading and in their absorptive qualities. For estimating purposes it is common to allow $\frac{1}{2}$ gal. per sq. yd. per in. of finished thickness, with 15 to 30% allowance for extra requirements. The quantity used generally falls within the range of the estimate. One group of engineers advocates the least quantity that will coat the particles; another advocates the largest quantity that can be incorporated without producing actual instability. The first group seemingly reasons that the surface area of particles is the significant factor, implying that the binding action of oil comes from surface tension, which, in turn, implies that the quantity of oil should not exceed that required to produce a film around each particle. The other group argues that the voids are most significant and must be filled with oil. In the meantime, errors in the quantity of oil applied during the first treatment are so readily corrected that the difference of opinion is not serious. However, it is obvious that the less oil used, the cheaper will be the construction cost, a statement which applies with particular force to projects distant from the source of oil supply.

Width, Thickness, and Cost

The tendency to use a minimum width of 20 ft. became marked during 1929. Likewise, a compacted thickness of 3 in. is becoming established. Few projects are being constructed less than $2\frac{1}{2}$ in. thick. There is some tendency to use a greater thickness than 3 in., especially on plant-mixed projects.

Cost of treatment exclusive of road metal continues to range rather widely from about $1 200 to as much as $2 000 per mile. The foregoing applies to averages. Individual projects within a State will vary even more widely, depending upon local conditions. The cost of oil is the largest variant. The cost of actual mixing, omitting the cost of oil and the application thereof, shows quite remarkable uniformity at $500, or less, per mile of 18-ft. road.

Failures

It is safe to state that the principal failures continue to arise from faulty support. Support may be deficient in thickness of metal or stability of sub-grade. As frequently stated, an impervious oil surface aggravates the effect of moisture in the sub-grade. Since many oil-treated projects are being undertaken upon metal insufficient for untreated conditions, it may be expected they will be found deficient when oil-treated, regardless of the method used. Emulsification of the light fuel oils in the presence of moisture and traffic is frequently reported. If the moisture comes from rainfall, the mixture can be protected by the application of a seal coat of heavy asphaltic oil and stone chips. Even a light application of the fuel oil used in the mixture, approximately $\frac{1}{8}$ gal. per sq. yd., covered with coarse sand, is commonly an effective protection against moisture and a preventative of raveling. Investigation shows that most cases are accompanied by a third condition, namely, the presence of an excessive amount of 200-mesh material, usually clay. Possibly the trouble may be avoided by eliminating clay from the mixture, which, however, is not always practical.

Rain during construction is a source of much annoyance. The immediate effect of free moisture is very similar to that of excess of oil, giving a deceiving appearance of richness and causing the material to gather in lumps while being mixed. If the moisture is retained after spreading, it produces waving and bleeding. The effect of moisture on the mixture during construction and afterward makes it advisable at the present stage of knowledge of the subject to proceed cautiously in moist climates. This statement is particularly true of plant mixtures, because a wet pit cannot be utilized without drying.

Emulsified and Cut-Back Binder

Emulsified asphalt was used for binder in road mixing, first, in 1928 and 1929. Two alternate methods have been used: First, water is spread upon the aggregate prior to and during the mixing to prevent the emulsion from breaking until mixed; and, second, a preliminary application of $\frac{1}{4}$ to $\frac{1}{2}$ gal. per sq. yd. of fuel oil is incorporated into the dry aggregate, improving the mixing qualities of the emulsion. A seal coat of about $\frac{1}{4}$ gal. of emulsion per sq. yd., covered with fine stone chips, is found advantageous to complete either method. Emulsified asphalt has been used in surface treatment with marked success. Cut-backs have also been used, although not extensively in the West.

Following experience with emulsions, it was suggested that cut-back asphalt or heavy road oil would be a suitable binder for a mixing treatment. This type of binder is accepted in California as having passed the experimental stage.

In using emulsions or cut-backs in road mixtures, it is hoped, of course, to produce a harder and more stable surface than can be attained with fuel oil. Engineers and highway authorities are not quite prepared to believe that an asphalt commonly designated "fuel oil" has sufficient binding prop-

erties to make a really successful pavement. Oil mixtures admittedly are soft and easily disturbed when first laid, whereas asphaltic concrete hardens at once; hence, it is reasoned that the early hardening of asphalt applied as an emulsion or a cut-back is highly desirable. The truth or error of this reasoning will not be known until a number of projects have been subjected to several years of service, after which a study of their construction and maintenance costs should furnish the answer.

In this connection, much should be learned from a test road in the Truckee River Canyon constructed in 1929 by the California Division of Highways. Thirteen road-mixed sections were built, which were followed in the spring of 1930 by a number of surface-treated sections. In the mixed sections two grades of fuel oil, two of cut-back, and two of emulsified asphalt were used, in combination with crushed gravel and crushed rock of varied grading. Accurate material and cost data were recorded. Although the combinations constitute only a few of the many possibilities, their behavior and the records of maintenance costs should be very informative, and the official reports will be of great interest when published.

New Construction

First undertaken as a maintenance measure, oil treatments of both types are beginning to have a profound effect upon design and new construction in the Western States. Few Western State highways carrying as many as 500 vehicles per day of summer traffic are being built without a definite program for oiling, and, in many States, the traffic limit is even less.

Approaching the use of oil from the construction standpoint, surface treatment has the disadvantage of requiring a well-compacted base, which ordinarily cannot be had without a tedious wait, or by using an undesirable clay binder. Road mixing requires special equipment that, ordinarily, contractors do not have, and surfacing is quite commonly finished during the season when oiling is impractical. Consequently, many States are restricting oil mixing to their own forces, even on new construction, but are adjusting the grading requirements of new surfacing material to the needs of oiling. Plant mixing has the advantage of being suitable for contract operations, which, perhaps, is one reason for its general adoption upon new projects in California.

The introduction of oiling has brought about a new examination of specifications for low and intermediate types of surfacing. Much material formerly considered too fine has become available. Binder, or filler, is being scrutinized more carefully. On the other hand, the possibilities of penetration macadam and the elimination of fine crushing are attracting attention. Thus far, the facility with which fine-crushed material may be manipulated with machinery, as contrasted with hand labor and other expensive operations for macadam, shows the oil mixture to be much the lower in price.

Oil treatments have never been seriously proposed as a substitute for standard higher types of pavements, but they are actually encroaching upon that field to some degree where freedom from dust and convenience to traffic

dominate the selection of type. Heretofore, for lack of a satisfactory or cheap intermediate type, roads carrying from 500 to 1 000 vehicles per day have frequently been paved, but oil treatment is now being found sufficient for at least immediate purposes. Thus, oil treatment is finding its place, mainly at the expense of the lower types, but to some degree as a substitute for the higher types.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

Discussion

BY MESSRS. GEORGE E. LARGE, AND SOPHUS THOMPSON AND R. W. CUTLER.

GEORGE E. LARGE,[27] ASSOC. M. AM. SOC. C. E. (by letter).[27a]—The writer believes the profession stands greatly indebted to Professor Cross for his development of the moment distribution method for analyzing continuous frames. A theoretically exact solution of many statically indeterminate structures can now be made with almost no mathematical drudgery. Moreover, the engineer can visualize the action of the structure while the solution is unfolding, and can thus develop a valuable sense of continuity seldom acquired in the use of calculus methods.

The writer has interested himself in a means of applying Professor Cross' method to structures in which the cross-section of a member varies along its length. Previous experience in analyzing and testing some such structures showed that an average value of the moments of inertia of a non-uniform member should not be substituted in the $\frac{I}{L}$ expression for stiffness of uniform members; neither are fixed-end moments and carry-over factors the same. Obviously, there is an infinite number of possible variations of the cross-section of members along their lengths. In comparatively few cases is it possible and feasible to write the equation of moment of inertia variation and perform the necessary integrations to find the required constants. The tables and diagrams prepared by some investigators cannot well cover all the possibilities.

NOTE.—The paper by Hardy Cross, M. Am. Soc. C. E., was published in May, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1930, by Messrs. C. P. Vetter, L. E. Grinter, S. S. Gorman, A. A. Eremin and E. F. Bruhn; October, 1930, by Messrs. A. H. Finlay, R. F. Lyman, Jr., R. A. Caughey, Orrin H. Pilkey, and I. Oesterblom; November, 1930, by Messrs. Edward J. Bednarski, S. N. Mitra, Robert A. Black, and H. E. Wessman; January, 1931, by Messrs. Jens Egede Nielsen, F. E. Richart, and William A. Oliver; February, 1931, by Messrs. R. R. Martel and Clyde T. Morris; March, 1931, by Francis P. Witmer, M. Am. Soc. C. E.; May, 1931, by Messrs. T. F. Hickerson, F. H. Constant, W. N. Downey and E. C. Hartman; and September, 1931, by Messrs. Thomas C. Shedd, David M. Wilson, and Marshall G. Findley.

[27] Asst. Prof. of Civ. Eng., and Research Engr., Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

[27a] Received by the Secretary August 31, 1931.

A workable method is presented herein which permits of any conceivable variation of the moment of inertia of a member. It merely requires that the moment of inertia variation be known. Having found the fixed-end moments,

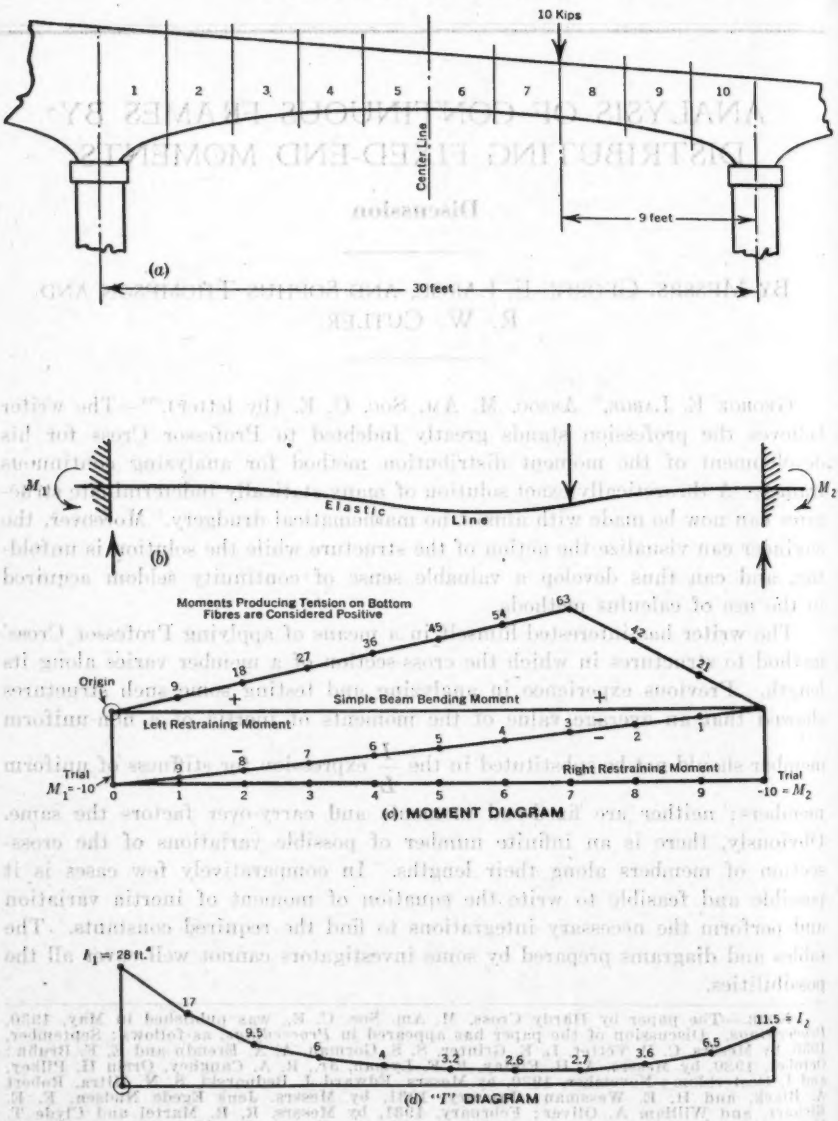


FIG. 47.

and the stiffness and carry-over factors by this method, the solution is completed by moment distribution.

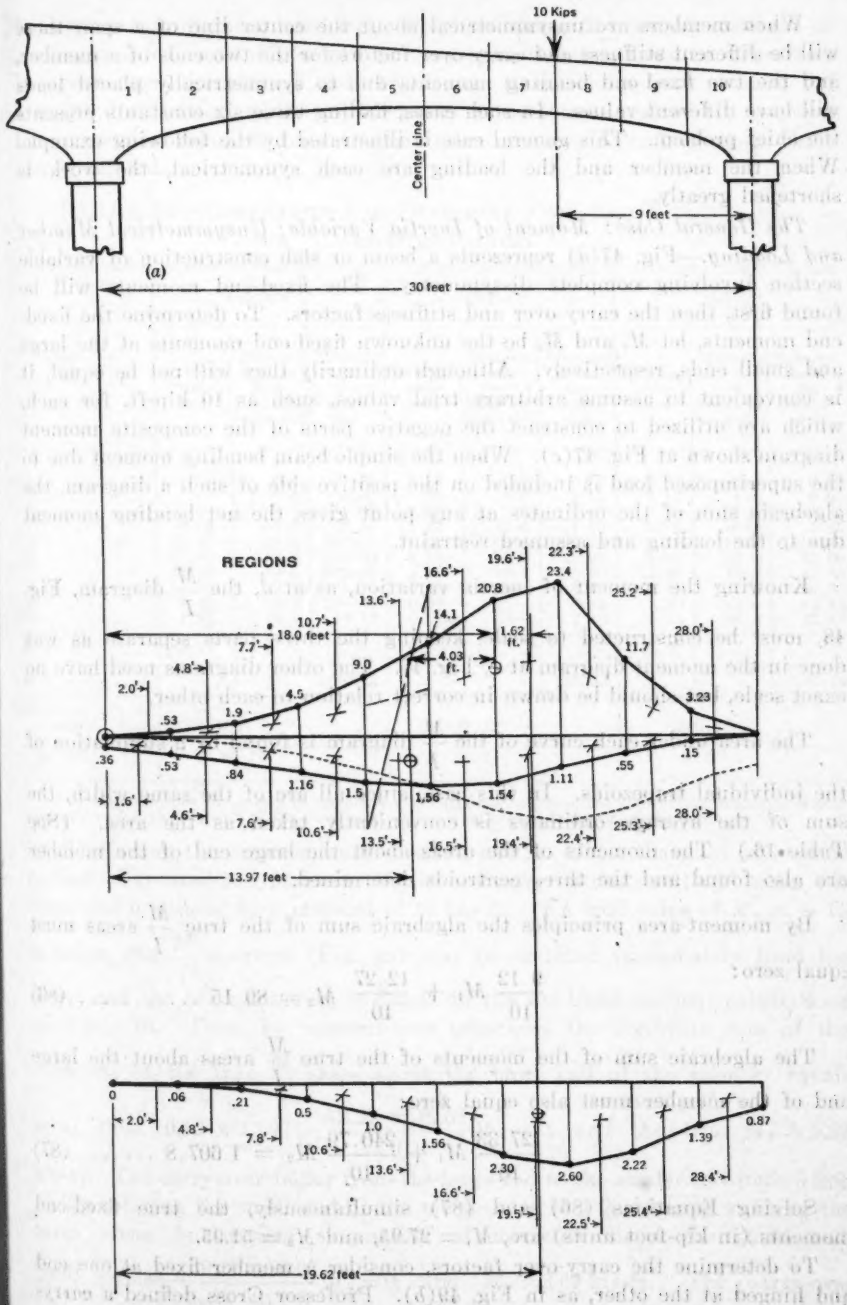


FIG. 48.—$\frac{M}{I}$ DIAGRAM.

When members are unsymmetrical about the center line of a span there will be different stiffness and carry-over factors for the two ends of a member, and the two fixed-end bending moments due to symmetrically placed loads will have different values. In such cases, finding these six constants presents the chief problem. This general case is illustrated by the following example. When the member and the loading are each symmetrical, the work is shortened greatly.

The General Case: Moment of Inertia Variable; Unsymmetrical Member and Loading.—Fig. 47(*a*) represents a beam or slab construction of variable section involving complete dissymmetry. The fixed-end moments will be found first, then the carry-over and stiffness factors. To determine the fixed-end moments, let M_1 and M_2 be the unknown fixed-end moments at the large and small ends, respectively. Although ordinarily they will not be equal, it is convenient to assume arbitrary trial values, such as 10 kip-ft. for each, which are utilized to construct the negative parts of the composite moment diagram shown at Fig. 47(*c*). When the simple-beam bending moment due to the superimposed load is included on the positive side of such a diagram, the algebraic sum of the ordinates at any point gives the net bending moment due to the loading and assumed restraint.

Knowing the moment of inertia variation, as at *d*, the $\frac{M}{I}$ diagram, Fig. 48, must be constructed to scale, keeping the three parts separate as was done in the moment diagram at *c*, Fig. 47. The other diagrams need have no exact scale, but should be drawn in correct relation to each other.

The area under each curve of the $\frac{M}{I}$ diagram is found by a summation of the individual trapezoids. In this case, since all are of the same width, the sum of the average ordinates is conveniently taken as the area. (See Table 16.) The moments of the areas about the large end of the member are also found and the three centroids determined.

By moment-area principles the algebraic sum of the true $\frac{M}{I}$ areas must equal zero:

$$\frac{9.12}{10} M_1 + \frac{12.27}{10} M_2 = 89.15 \quad \text{.....(86)}$$

The algebraic sum of the moments of the true $\frac{M}{I}$ areas about the large end of the member must also equal zero:

$$\frac{127.33}{10} M_1 + \frac{240.79}{10} M_2 = 1\,607.8 \quad \text{.....(87)}$$

Solving Equations (86) and (87) simultaneously, the true fixed-end moments (in kip-foot units) are, $M_1 = 27.95$, and $M_2 = 51.95$.

To determine the carry-over factors, consider a member fixed at one end and hinged at the other, as in Fig. 49(*b*). Professor Cross defined a carry-over factor as the ratio of the moment at the fixed end to the moment pro-

ducing rotation at the rotated end. For members with constant moments of inertia, it is 0.5, but for non-uniform members its values must be calculated. In every case, the rotated end corresponds to the joint just released for moment distribution, while the fixed end is the remote end of the member. Unsymmetrical members have two carry-over factors, depending upon which end is rotated.

TABLE 16.—COMPUTATIONS TO DETERMINE FIXED-END MOMENTS AND CARRY-OVER FACTORS

Piece No. (Fig. 47)	Positive $\frac{M}{I}$			Trial $\frac{M_1}{I}$			Trial $\frac{M_2}{I}$		
	Average ordinate	Moment arm, in feet, from the large end	Moment about the large end	Average ordinate	Moment arm, in feet, from the large end	Moment about the large end	Average ordinate	Moment arm, in feet, from the large end	Moment about the large end
1	0.265	2.0	0.53	0.45	1.6	0.72	0.03	2.0	0.06
2	1.21	4.8	5.80	0.68	4.6	3.13	0.13	4.8	0.62
3	3.20	7.7	24.60	1.00	7.6	7.60	0.36	7.8	2.81
4	6.75	10.7	72.30	1.33	10.6	14.10	0.75	10.6	7.95
5	11.55	13.6	157.00	1.53	13.5	20.70	1.28	13.6	17.40
6	17.45	16.6	290.00	1.55	16.5	25.60	1.93	16.6	32.00
7	22.10	19.6	433.00	1.32	19.4	25.60	2.45	19.5	47.85
8	17.55	22.3	391.50	0.83	22.4	18.93	2.41	22.5	54.30
9	7.46	25.2	188.00	0.35	25.3	8.85	1.80	25.4	45.70
10	1.61	28.0	45.10	0.075	28.0	2.10	1.13	28.4	32.10
Area* $\propto$	89.15	.....	.....	9.12	.....	.....	12.27	.....	.....
$M \propto$	.....	.....	1 607.83	.....	.....	127.33	.....	.....	240.79
Distance of the centroid from the large end, in feet	$\frac{1607.8}{89.15} = 18.00$			$\frac{127.33}{9.12} = 13.97$			$\frac{240.8}{12.27} = 19.62$		

* In computations for fixed-end moments and carry-over factors, the sum of the ordinates is conveniently called the "area" of the diagrams. The actual area is three times the value given.

Continuing the example, and referring to Fig. 49(*b*), the chief problem is to find what fixed-end moment, M_B, is necessary at the small end when the large end is rotated by a moment of 10 kip-ft. If a trial value of $M_B = -10$ is taken, the $\frac{M}{I}$ diagram (Fig. 48) may be sketched immediately from the areas, and the centroids may be found in the fixed-end moment calculations of Table 16. Then, by moment-area principles the algebraic sum of the moments of the true $\frac{M}{I}$ areas about the large end of the member equals zero; thus, $9.12 \times 13.97 - \frac{12.27 \times 19.62}{10} M_B = 0$, and the true $M_B = 5.29$ kip-ft. The carry-over factor from the large end to the small end equals 0.529.

By rotating the small end of the member, as in Fig. 50, and utilizing the same areas, but taking summation of moments about the small end, $12.27 \times 10.38 - \frac{9.12 \times 16.03}{10} M_D = 0$, and $M_D = 8.69$ kip-ft. The carry-over factor from the small end to the large end equals 0.869.

Stiffness Factor.—The stiffness of a member is measured by the moment necessary to turn the hinged end of a member through a unit angle when the other end is fixed. When a joint is released in the moment-distribution

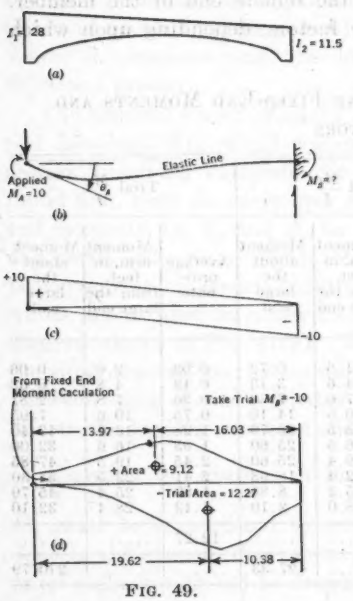


FIG. 49.

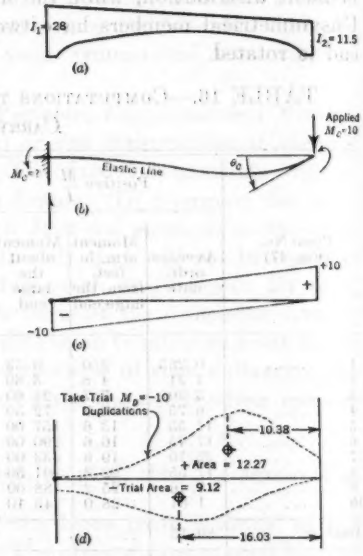


FIG. 50.

method, all the ends of the members connecting directly thereto are turned through the same angle. Of the entire moment to be distributed, the part that each member assumes is proportional to its stiffness.

It is proved in mechanics that, for a uniform member, $E\theta_A = \frac{M_A L}{4I}$, or $M_A = \frac{4 E I \theta_A}{L}$. (See Fig. 49.)

Since, at any given joint, E and θ_A are constant for all members connecting thereto, the moment upon each will be proportional to its ratio, $\frac{I}{L}$. Accordingly, $\frac{I}{L}$ is used to state the relative stiffnesses of the uniform members of a structure, and is called the stiffness factor.

Since a structure may consist of both uniform and non-uniform members, it is necessary to state the stiffness factors for the non-uniform members in terms of $\frac{I}{L}$ also; thus,

$$S = \frac{k I_0}{L} \qquad (88)$$

in which, S is the stiffness factor for a non-uniform member; k is a constant

to be determined for each member; and I_0 is the moment of inertia at a particular section of reference.

Furthermore, for non-uniform members,

$$E\,\theta_A = \frac{M_A\,L}{4\,k\,I_0} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (89)$$

Referring again to Fig. 49, one may state concerning the algebraic sum of the positive and true negative areas, by moment-area principles: $E\theta_A = 3\,(9.12 - 0.529 \times 12.27) = \dfrac{10 \times 30}{k_1 \times 4 \times 28}$, using 3 to state the real area (see Table 16) and the largest moment of inertia, $I_1 = 28$, for reference Finally, $k_1 = 0.34$ and the stiffness factor for the large end $= 0.34\,\dfrac{I_1}{L}$.

Similarly, for the small end (see Fig. 50): $E\theta_C = 3\,(12.27 - 0.869 \times 9.12) = \dfrac{10 \times 30}{k_2 \times 4 \times 28}$. Again, using the largest moment of inertia as a reference, $k_2 = 0.205$ and the stiffness factor for the small end $= 0.205\,\dfrac{I_1}{L}$.

Model Check of Stiffnesses Found.—In order to check the stiffnesses of the two ends of the member as computed herein, a model was made from $\frac{1}{16}$-in. sheet celluloid embodying the exact moment of inertia variation used in the

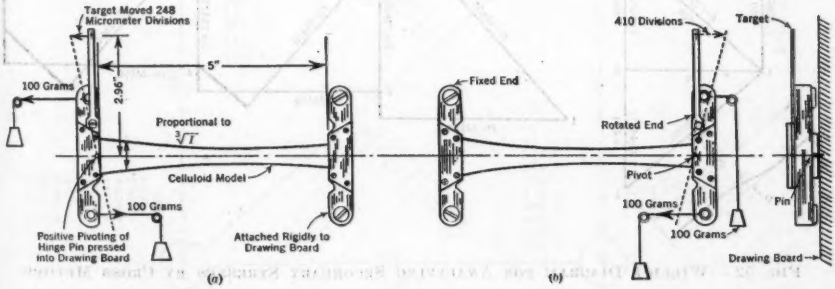


FIG. 51.

calculations. A bar of a Beggs deformeter gauge was attached to each end of the model, as shown in Fig. 51. One bar was fixed to a drawing-board with screws, while the other was free to rotate about its hinge-pin which had been pressed into the drawing-board. A known torque was applied to the pivoted bar and the displacement of a target was observed with a micrometer microscope. The model was then reversed, and the other end rotated under the same torque.

The average displacements were 248 and 410 micrometer divisions for the rotation of the large and small ends, respectively. This indicates that the actual stiffness of the large end is $\dfrac{410}{248} = 1.652$ times that of the small

end. The corresponding ratio of the k's found in calculating the stiffnesses, is $\frac{0.340}{0.205} = 1.658$. This remarkable correlation also verifies the values of the carry-over factors since the stiffnesses were calculated therefrom.

SOPHUS THOMPSON[28] and R. W. CUTLER,[29] JUNIORS, AM. SOC. C. E. (by letter).[29a]—In this paper, mention was made of the application of the moment distribution method to the solution of secondary stresses in a truss. The writers have made a study of this phase. For the sake of simplicity and comparison of results, the truss analyzed herein has been chosen from among those treated in current textbooks by the "exact" method.

The method used by the writers requires the construction of the Williot diagram (Fig. 52), from which the value of D for each member is obtained, D being the displacement, at right angles to the axis of the member, of one end with respect to the other. By cutting each member at the mid-point and

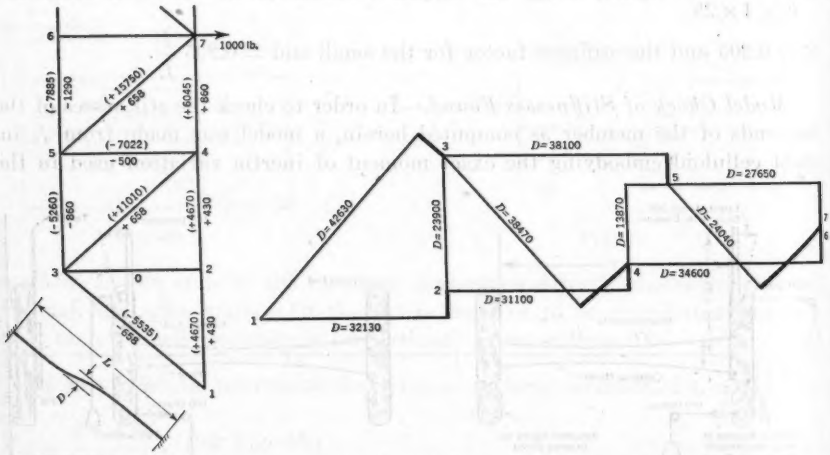


FIG. 52.—WILLIOT DIAGRAM FOR ANALYZING SECONDARY STRESSES BY CROSS METHOD.

treating each half as a cantilever with the deflection, $\frac{1}{2}D$, the fixed-end moments are obtained. The value of D as used herein is actually displacement multiplied by E. Then, referring to Fig. 53, $\frac{D}{2} = \frac{PL^3}{3I} = \frac{ML^2}{3I}$, or $M = \frac{3ID}{2L^2}$.

The determination of the algebraic sign of the fixed-end moments is simplified, first, by the use of the convention which designates as positive any moment tending to rotate a joint in the clockwise direction; and, second, by considering the displacement of the ends of a member relative to each other, as shown on the Williot diagram. For drawing the Williot diagram, Member 6–7 is assumed to stand fast.

[28] Asst. Prof. of Civ. Eng., Southern Methodist Univ., Dallas, Tex.
[29] Dallas, Tex.
[29a] Received by the Secretary October 5, 1931.

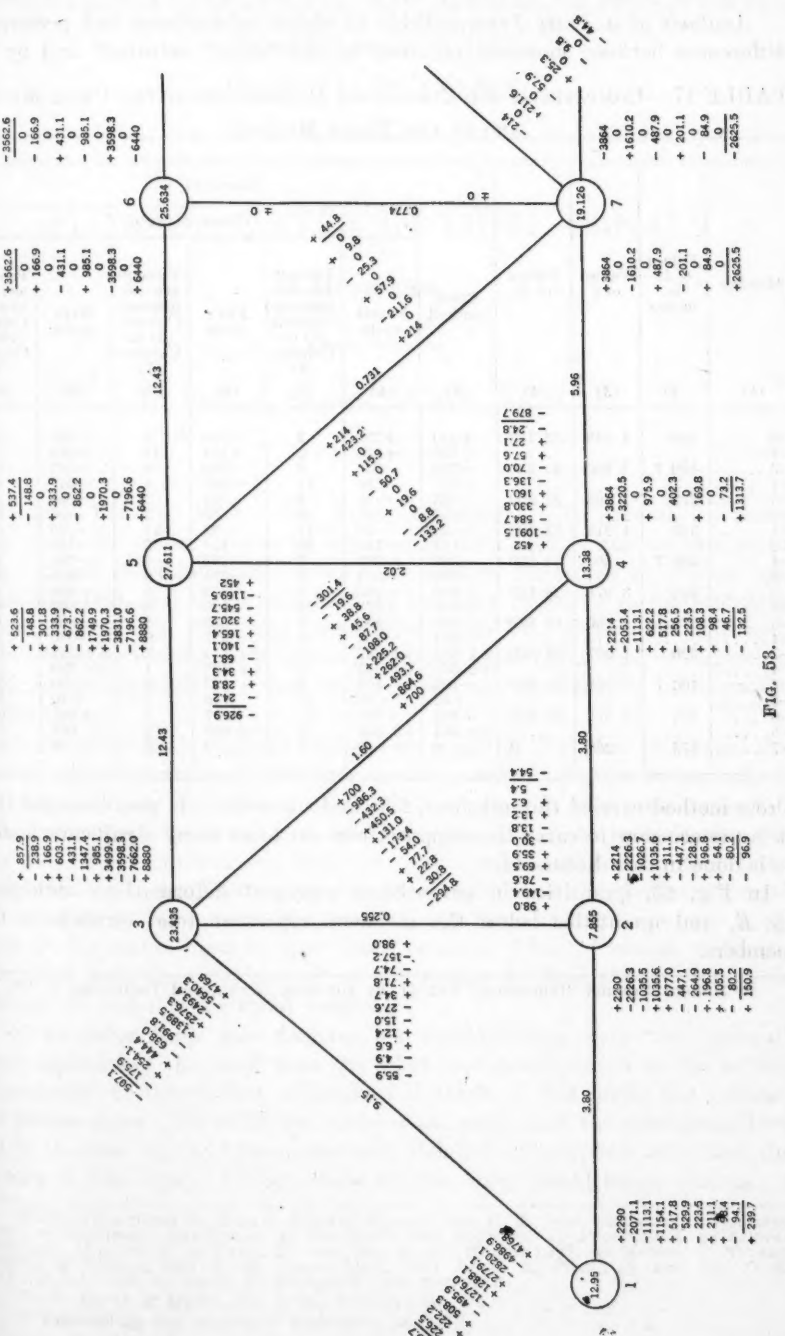


FIG. 53.

Analysis of a Pratt Truss.—Table 17 shows comparisons and percentage differences between moments obtained by the "exact" solution[30] and by the

TABLE 17.—COMPARISONS AND PERCENTAGE DIFFERENCES BY THE CROSS METHOD AND BY THE EXACT METHOD

Member	Values of L, in inches	Values of I	Values of D	Moments						
				Exact method	Cross Method					
					Fourth cycle	Percentage difference, Column (6) to Column (5)	Fifth cycle	Percentage difference, Column (8) to Column (5)	Sixth cycle	Percentage difference, Column (10) to Column (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1–2	320	1 218	32 130	+230	+235	2	+240	4	+237	3
2–1	.....	.....	.....	+135	+126	6	+151	12	+139	3
1–3	490.7	4 490	42 630	−230	−235	2	−240	4	−237	3
3–1	.....	.....	.....	−527	−586	11	−507	4	−542	3
2–3	372	95	23 900	−53	−55	4	−54	2	−54	2
3–2	.....	.....	.....	−55	−57	4	−56	2	−56	2
2–4	320	1 218	31 100	−80	−70	12	−97	21	−85	6
4–2	.....	.....	.....	−141	−184	30	−133	6	−153	8
3–4	490.7	805	38 470	−307	−287	7	−295	4	−291	5
4–3	.....	.....	.....	−320	−321	3	−301	6	−309	3
3–5	320	3 978	38 100	+900	+930	3	+858	5	+889	1
5–3	.....	.....	.....	+495	+370	25	+523	6	+463	6
4–5	372	750	13 870	−875	−882	1	−880	½	−882	1
5–4	.....	.....	.....	−924	−931	1	−927	0	−930	1
4–7	320	1 907	34 600	+1 342	+1 387	3	+1 314	2	+1 344	0
7–4	.....	.....	.....	+2 602	+2 541	2	+2 625	1	+2 589	½
5–7	490.7	358	24 040	−130	−124	5	−133	2	−130	0
7–5	.....	.....	.....	+42	+35	17	+45	7	+40	5
5–6	320	3 978	27 650	+562	+685	22	+537	4	+597	6
6–5	.....	.....	.....	+3 544	+3 396	4	+3 563	½	+3 488	2
6–7	372	288	0	0	0	0	0	0	0	0

Cross method carried through four, five, and six cycles. It may be noted that it is unnecessary to carry the computations out to as many significant figures as is done in this discussion.

In Fig. 53, quantities in parentheses represent deformations multiplied by E, and quantities below the members represent total stresses in the members.

[30] "Modern Framed Structures," Vol. II, by Johnson, Bryan, and Turneaure, p. 440.

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DISCUSSIONS

FLOW OF WATER IN TIDAL CANALS

Discussion

BY EARL I. BROWN, M. AM. SOC. C. E.

EARL I. BROWN,[41] M. AM. SOC. C. E. (by letter).[41a]—In general, the reception which the theory of flow in tidal canals, advanced in the writer's paper, has received, has been quite cordial. No serious differences of opinion have been expressed, and no serious question has been raised as to the soundness of the theory. Mr. Faris and Commander Rude have given some additional information to clarify parts of the paper, but those parts were in connection with statements made to round out the paper, not affecting the fundamentals of the theory.

Commander Rude suggests that the propagation of a wave in a canal that has an outlet into a body of water would continue into the outlet body in its original form, and that it would be dissipated in that body instead of being propagated backward as a reflected wave. It is the writer's view that the wave does continue out into the outlet body, but not in its original amplitude. The amplitude must change in accordance with the law expressed by Equation (37), in which, if l_1 becomes very large or infinite, h_1, the amplitude at the outlet, must be very small or zero. This, of course, is because the canal wave does not possess enough energy to raise a wave in a wider expanse of water to an equal height.

Commander Rude also declares his understanding that "the hydraulic characteristics of the canal were not taken into consideration by the writer." Apparently, he means that no account is taken of flow under the influence of surface slope. When the two ends of the canal have the same mean level, as is the case in the Chesapeake and Delaware Canal, the only flow that occurs is tidal flow. If there were no tide there would be no current. A

NOTE.—The paper by Earl I. Brown, M. Am. Soc. C. E., was published in December, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: February, 1931, by R. L. Faris, M. Am. Soc. C. E.; March, 1931, by Messrs. G. T. Rude, Eugene E. Halmos, and W. M. Black; May, 1931, by H. F. Flynn, M. Am. Soc. C. E.; and October, 1931, by Ralph G. Davies, M. Am. Soc. C. E.

[41] Col., Corps of Engrs., U. S. A., Philadelphia, Pa.

[41a] Received by the Secretary September 26, 1931.

discussion is given under the heading, "Insufficiency of Theory That Flow Caused by Surface Slope Accounts for Tidal Flow," to show how a permanent regimen of flow under difference of tidal level at the two ends of the canal can never become established. The slope is constantly variable, and there is never time for such a regimen to become established; hence, there is no such flow. The flow that does exist tends to approach such a regimen, but its velocity is measured by the expression given in either Equation (15) or Equation (50), depending on whether the wave is simple or compound. It is because of the tendency of the current to approach the regimen of flow under the influence of surface slope that makes it possible, as Commander Rude states, to predict currents at the railroad bridge approximately on that assumption. The fact that this can be done at the railroad bridge is a fortuitous chance, however, as it does not follow by any means that it could be done elsewhere in the canal.

Commander Rude expresses a doubt as to whether the writer accepts the "'stationary wave theory' of Harris as an explanation of the changed regimen of the tide in certain bodies of water in which the tide occurs almost simultaneously throughout the entire length of the body * * *."

The "stationary wave" of Harris is nothing more than a resultant wave formed by the interference of two equal and opposed waves (see heading, "Theory of the Interference of Two Opposed Waves"). An illustration of it occurs when a simple wave of translation is propagated into a canal of fairly uniform cross-section, closed at one end, and of length just equal to one-half the wave length. (The positive and negative phases of the ordinary tidal wave are considered as separate waves in this theory.) The canal will then have a stationary wave with a regimen exactly as Harris defines it. Such a wave is known to exist in certain bays such as Long Island Sound, where the western end is almost closed, and in certain seas, where the depth and superficial dimensions bear the proper relationship to the period of the tide for reflections from its shores to interfere with the oncoming wave in the proper phase relationship to create the stationary wave. Harris' theory is, therefore, a special case of the more general theory of reflected waves in interference.

Replying to Commander Rude's inquiry as to Tables 3, 4, and 5, it may be stated that the data in the columns headed, "Bourdelles," are based on observation. Those in columns headed, "Levy," are values computed by Levy according to his formulas.

In commenting on the differences between the Levy formulas as presented by Mr. Parsons, and those of the writer, Mr. Halmos states:

> "The derivation of the [Parsons'] formulas is strictly rational, their application being restricted only by the requirement of comparatively small tidal variations in relation to the depth of the canal."

This restriction, by the requirement of comparatively small tidal variations in relation to the depth of the canal, is sufficient to exclude these formulas from use in most artificial tidal canals constructed for navigation, in which the depth will probably seldom exceed 35 ft., and in which tidal

variations of 4 to 5 ft. must be expected. Variations of these magnitudes cannot be considered as small with respect to the depth. In addition to this defect, the writer urges another. The expressions, P and Q, in Equations (2) of the paper[42] by Mr. Parsons, are assumed to be logarithmic functions of x in the formulas, $P = AC^{ax}$, and $Q = BC^{ax}$. It is the writer's opinion that P and Q are not only functions of x, but also of h, which is the quantity sought to be determined. It is the failure of the formulas to include h in the values of P and Q which makes them give equal values for positive and negative waves.

Mr. Halmos suggests that the formulas of the French Academy of Sciences recommended by Mr. Parsons for canals of certain characteristics might have been useful in predicting approximate values of heights and velocities. Their use was rejected by the writer because they are based on the assumptions of a uniform slope of the water surface from end to end of the canal and of the velocity varying with that surface slope, both of which he holds to be erroneous.

The writer admits the difficulty apprehended by Mr. Halmos in finding a proper value of F by the proposed method. Its value for any proposed canal can be estimated with confidence only when it has actually been determined experimentally from existing similar canals.

General Black points out how the useful range of the theory may be extended to the study of the effects of changes projected in the cross-section of tidal canals, and cites the control of current velocities in the East River, at New York City, as an example. The writer may venture further and call attention to the way in which control of these current velocities might be obtained directly by the use of this theory.

The tide in Long Island Sound is of the stationary type due to the reflection of a wave back on itself from the nearly closed western end. The height of the tide increases at a fairly uniform rate from east to west. The depth of the Sound and its length have the required relationship with the tidal period to make the tidal phenomena almost a perfect example of the stationary type of tide. If this relationship were destroyed, the excessive amplitude at the point of reflection near Throgs Neck would be lessened, and this, in turn, would reduce the East River currents. The relationship could be destroyed by making one or more artificial contractions of Long Island Sound at proper intervals in the distance from Fishers Island to Throgs Neck, breaking up the wave by reflections having a less favorable relationship to cause high amplitudes. A thorough study of this problem by the aid of this theory, it is believed, could lead to a solution giving any desired control of the East River currents.

General Black's hope that similar studies will be made of the tidal action in estuaries such as the Hudson and Delaware Rivers will be realized in the case of the Delaware River. This study is now (1931) in progress.

[42] "The Cape Cod Canal," by William Barclay Parsons, Hon. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 111.

The readiness with which the effect of changed conditions in the canal are reflected in the results of computations is indicated in the curves given by Messrs. Flynn and Davis. Mr. Flynn showed that a material reduction of discrepancies between observed and computed results could probably be made by a revision of preliminary assumptions to conform more nearly with actual conditions. Mr. Davis has recomputed predicted results under revised assumptions corresponding to actual conditions. The close agreement of his computed and observed results is very gratifying.

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DISCUSSIONS

THE DON MARTIN PROJECT

Discussion

BY ANDREW WEISS, M. AM. SOC. C. E.

ANDREW WEISS,[8] M. AM. SOC. C. E. (by letter).[8a]—The writer is indebted to Mr. Howell for the pertinent explanatory comments, stating more fully the reasons for the adoption of the buttress type of dam.

To date (1931) only two of the contact joints between the buttresses have shown leakage and this in negligible quantities. As the water was being raised within the reservoir, some leakages developed in varying amounts through the limestone seams around the south abutment, and these were effectually followed by drilling and grouting. At this writing, the reservoir is filled to within 3 ft. of the flowage line.

The automatic opening feature on all the radial gates was further tested, with satisfactory results, by pumping water into the float wells, but incident to these operations a number of unforeseen details were discovered, which led to the adoption of certain further precautionary measures. One of these was the use of covers for the tubes admitting water to the float chambers, thus preventing careless or mischievous introduction of waste matter and obstacles. Some of the chains designed to connect the gates with the movable emergency lifting devices had become loosened from the upper corners of the gates and, with the automatic lift in operation, the suction of the escaping water had a tendency to drag the slack portion of these chains between the radial gate seats and the rubber seals, thus interfering with free movement. Hence, it was necessary to be certain of the locking of these latches which fasten the chains to the upper corners of the gates in order to avoid a recurrence of these happenings (see Fig. 13).

A view of the overflow section, looking up stream from the south end, is shown in Fig. 14. Fig. 15 shows the discharge of three gates under a 9-ft. head, each releasing about 2 200 cu. ft. per sec. It will be seen that immedi-

NOTE.—The paper by Andrew Weiss, M. Am. Soc. C. E., was published in December, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: March, 1931, by C. H. Howell, M. Am. Soc. C. E.; and May, 1931, by Robert S. Stockton, M. Am. Soc. C. E.

[8] Res. Engr., J. G. White Eng. Corporation, Estacion La Cruz, Chihuahua, Mexico.

[8a] Received by the Secretary September 16, 1931.

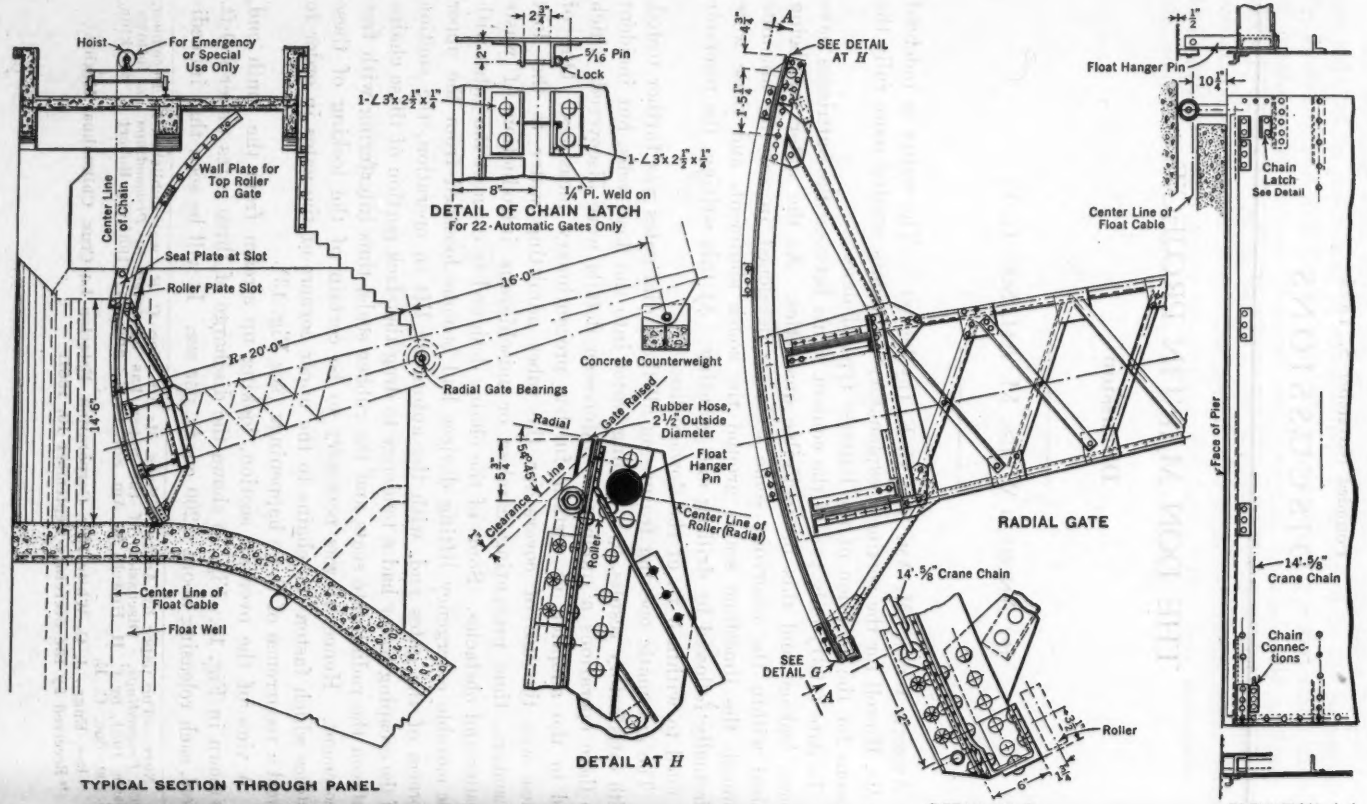


FIG. 13.—ASSEMBLY AND DETAILS OF CRANE CHAIR CONNECTIONS, RIO SALADO DAM

FIG. 15.—

FIG. 14.—

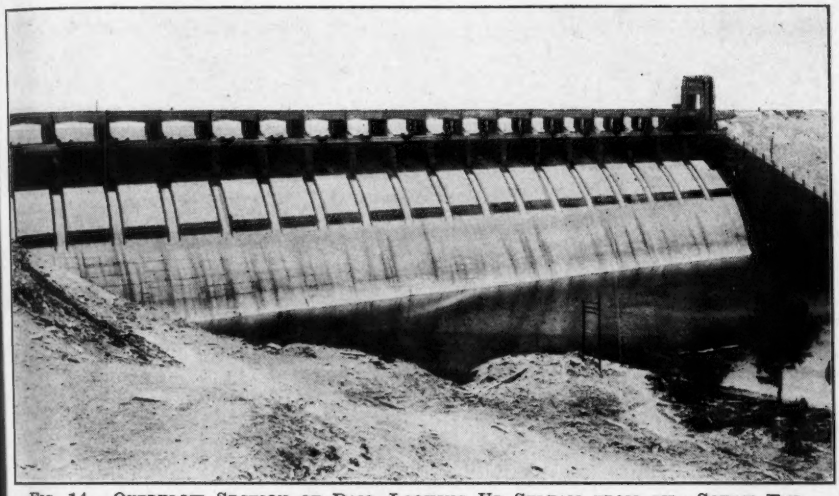


FIG. 14.—OVERFLOW SECTION OF DAM, LOOKING UP STREAM FROM THE SOUTH END.

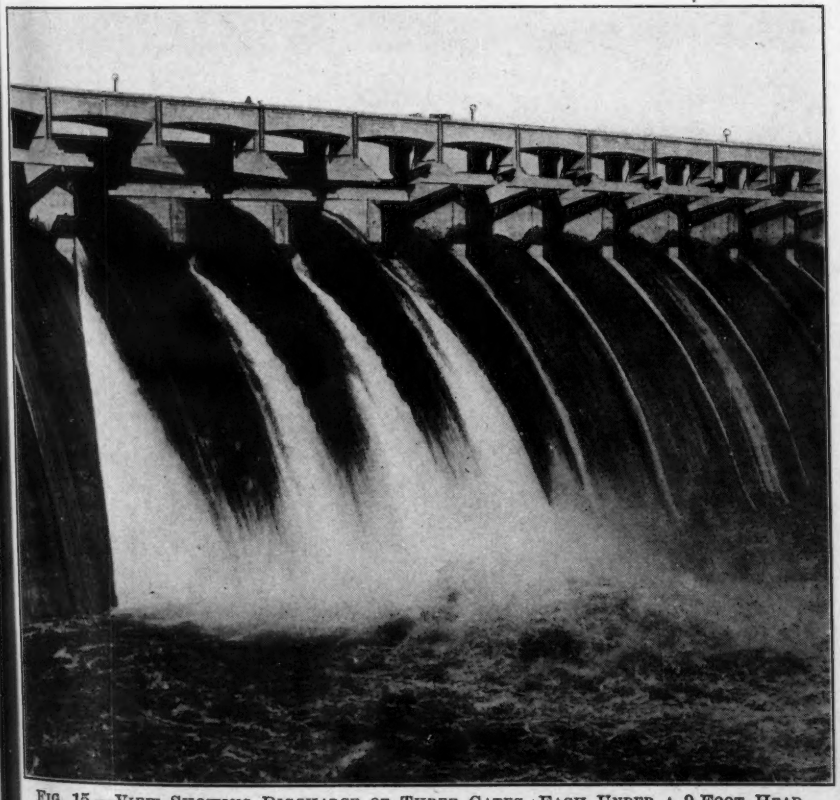


FIG. 15.—VIEW SHOWING DISCHARGE OF THREE GATES, EACH UNDER A 9-FOOT HEAD.

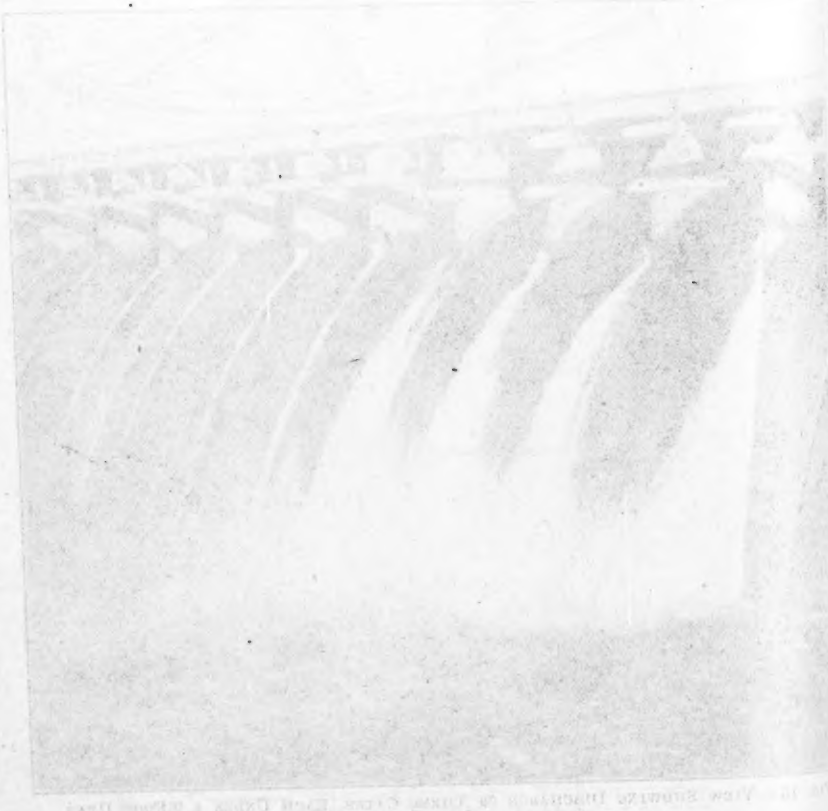
FIG. 13.—ASSEMBLY AND DETAILS OF CRANE CHAIR CONNECTIONS, RIO SALADO DAM

TYPICAL SECTION THROUGH PANEL

DETAIL AT H

DETAIL AT G

ELEVATION A-A



ately at the toe of the dam some spray and a turbulent wave action are formed; the latter, however, gradually reduces to a steady or normal flow condition about 20 ft. down stream from the dentated sill. With such quantities of water as have been released it is too soon to state how effective these dentated sills may be, but it appears that erosion below the toe will be kept entirely within harmless limits.

Mr. Stockton correctly points out the difficulties of obtaining settlers of the required financial capacity and training in agricultural pursuits, who will be able to put the lands on a productive basis before the interest and carrying charges have mounted to a point where it is no longer possible to collect them. This applies with particular emphasis to the new projects being built by the Mexican Federal Commission of Reclamation because of the greater scarcity of settlers with the requisite means. For this purpose the framers of the reclamation policy and the regulations expressing it, have wisely provided for three-year rental contracts and for farm supervision. It is too soon to predict the extent to which this supervision may be carried, but judging from the interest thus far displayed by the governmental authorities in charge, it is believed that such assistance will be freely provided and that the settlers will co-operate in a most responsive spirit.

Beginning with the very initiation of these reclamation schemes the original Commission and its successors in charge have given the most detailed study to the reclamation problems in the United States and elsewhere, and have familiarized themselves with the principal results of the many investigations during the course of development—more especially with the report of the Fact-Finding Commission—to the end that errors might be minimized. One of the outgrowths of this study was the conclusion that most of these projects could not be undertaken without some form of subsidy from the Federal Treasury and, for this reason, it was decided to fix the construction charges commensurately with the market value of the land in question. Doubtless much will have to be added to the present regulations as experience and economic developments may dictate within the coming years. It can not be over-emphasized, however, that the definite fixing of the construction charges corresponding to the market values of the land rather than reclamation costs has obviated much of the criticisms and incentives for attack on the Mexican Federal Commission on Reclamation and its engineers such as was practiced in the United States with the beginning of the repayments. This train of events was pointed out exceedingly well by F. H. Newell, M. Am. Soc. C. E., in his discussions of "Some Phases of Irrigation Finance."[9]

Mr. Stockton correctly indicates the importance of proper supervision and layout of farm management and that such be continued until a substantial equity in the holding has been established. In this case the regulations provide that the settler follow the general guidance of the farm advisers and the experiment station, from which sources it is believed much good will result. Much misunderstanding has existed as to the proper functions,

[9] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), pp. 563–568.

responsibilities, and results of agricultural advisers on these projects. It is pointed out with reason that no one man can combine the necessary experience and training to direct the farming operations of all the settlers. This is not and should not be expected. His functions and work are more nearly comparable with that of the field men as they are employed and directed by the sugar companies operating in the Western States. Few of these men would qualify as master farmers. They do fill an important need, however, as general advisers and as purveyors of information as well as keepers of statistical data relating to the needs and the prospects of the individual settlers. These data may furnish a reliable basis for the future credits and assistances that should be provided by the Government, or by other credit agencies.

Such a system, if properly managed and maintained, will in no sense impair the settler's individuality and self-reliance, as it is essentially co-operative and helpful. It is obviously unpopular with a certain minority of settlers and sympathizers who are chiefly interested in building and collecting pretexts and arguments in defense of non-payment and non-performance of just and practicable dues; but this fact does not argue against such a system of vigilance and help because the majority of the settlers on any project are essentially as honest and as anxious to meet their debts as they become due, as people are in other walks of life. Experience has shown that such a system of supervision soon becomes popular rather than otherwise if backed by the proper management and ideals.

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DISCUSSIONS

DESIGN OF A REINFORCED CONCRETE SKEW ARCH

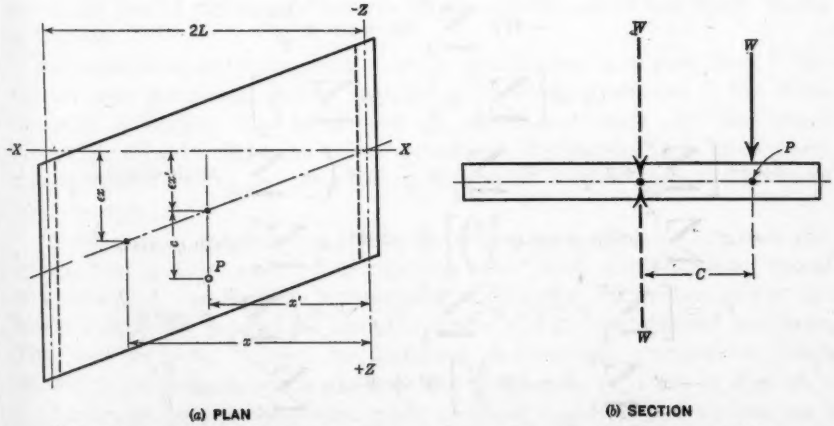
Discussion

BY A. A. EREMIN, ASSOC. M. AM. SOC. C. E.

A. A. EREMIN,[14] ASSOC. M. AM. SOC. C. E. (by letter).[14a]—The author has presented an extremely interesting paper on the design of a skew arch and "rigid frame." He has simplified the equations originally developed[15] by J. Charles Rathbun, M. Am. Soc. C. E., and has illustrated their application by a practical design problem.

The equations for a skew arch with the loads symmetrical about the center line between spandrel walls, are discussed in this paper. For the stresses in a skew arch with unsymmetrical concentrated loads the equations will be slightly different.

Fig. 37(*a*) is the plan of a skew arch with a concentrated load, *W*, at the point, *P*. The eccentricity is designated by *c*. The elevation of the arch is



(a) PLAN (b) SECTION

FIG. 37.—DIAGRAM SHOWING POSITION OF UNSYMMETRICAL CONCENTRATED LOAD.

NOTE.—The paper by Bernard L. Weiner, Assoc. M. Am. Soc. C. E., was published in January, 1931, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: May, 1931, by Messrs. G. D. Houtman, and J. Charles Rathbun.

[14] Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.

[14a] Received by the Secretary, June 29, 1931.

[15] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

the same as that shown in the author's Fig. 9(*b*). This unsymmetrical load may be replaced by a symmetrical load, W, and a moment, W_c. Then the equations for the moments will be:

$$m_u = R_x \epsilon x \cos\phi + R_y \epsilon x \sin\phi + R_z v' - N_x \sin\phi + N_y \cos\phi - W \epsilon (x - x') \sin\phi - W_c \sin\phi \quad \ldots\ldots (75d)$$

$$m_v = -R_x \epsilon x \sin\phi + R_y \epsilon x \cos\phi - R_z u' - N_x \cos\phi - N_y \sin\phi - W \epsilon (x - x') \cos\phi - W_c \cos\phi \quad \ldots\ldots (75e)$$

and,

$$m_z = R_x y + R_y x + N_z - W (x - x') \quad \ldots\ldots (75f)$$

The reactions at the arch may be obtained from the final equations:

$$-K_1 R_x + K_2 R_y + K_3 R_z + K_4 N_x - K_5 N_y = C_1 \quad \ldots\ldots (76a)$$

$$K_6 R_x - K_7 R_y + K_8 R_z + K_5 N_x + K_9 N_y = C_2 \quad \ldots\ldots (76b)$$

$$K_{10} R_x + K_{11} R_y + K_{12} N_z = C_3 \quad \ldots\ldots (76c)$$

$$K_{13} R_x + K_{14} R_y + K_{15} R_z + K_7 N_x + K_6 N_y + K_{10} N_z = C_4 \quad \ldots\ldots (76d)$$

$$K_{14} R_x + K_{17} R_y - K_{18} R_z - K_2 N_x - K_7 N_y + K_{11} N_z = C_5 \quad \ldots\ldots (76e)$$

$$-K_{15} R_x + K_{18} R_y + K_{19} R_z + K_3 N_x - K_8 N_y = C_6 \quad \ldots\ldots (76f)$$

in which, the coefficients, K, are as given by the author, in Equations (25), Appendix I, and the coefficients, C, are:

$$C_1 = \epsilon \left[\sum_A^P x \cos^2\phi \frac{\lambda s}{F} - x' \sum_A^P \cos^2\phi \frac{\lambda s}{F} \right] W + Wc \sum_A^P \cos^2\phi \frac{\lambda s}{F}$$

$$C_2 = \epsilon \left[-\sum_A^P x \sin\phi \cos\phi \frac{\lambda s}{F} + x' \sum_A^P \sin\phi \cos\phi \frac{\lambda s}{F} \right] W - Wc \sum_A^P \sin\phi \cos\phi \frac{\lambda s}{F}$$

$$C_3 = \left[\sum_A^P x \frac{s}{I} - x' \sum_A^P \frac{s}{I} \right] W$$

$$C_4 = \left[\sum_A^P x y \frac{s}{I} - x' \sum_A^P y \frac{s}{I} + \epsilon^2 \left(-\sum_A^P x^2 \sin\phi \cos\phi \frac{\lambda s}{F} + x' \sum_A^P x \sin\phi \cos\phi \frac{\lambda s}{F} \right) \right] W - W c \epsilon \sum_A^P x \sin\phi \cos\phi \frac{\lambda s}{F}$$

$$C_5 = \left[\sum_A^P x^2 \frac{s}{I} - x' \sum_A^P x \frac{s}{I} + \epsilon^2 \left(\sum_A^P x^2 \cos^2\phi \frac{\lambda s}{F} - x' \sum_A^P x \cos^2\phi \frac{\lambda s}{F} \right) \right] W + W c \epsilon \sum_A^P x \cos^2 \frac{\lambda s}{F}$$

$$C_6 = \epsilon \left[\sum_A^P u' x \cos\phi \frac{\lambda s}{F} - x' \sum_A^P u' \cos\phi \frac{\lambda s}{F} \right] W + W c \sum_A^P u' \cos\phi \frac{\lambda s}{F}$$

The notation and initial assumptions are the same as those used by Mr. Weiner. If the eccentricity is zero, the coefficients, C, will be the same as given by the author in Equation (25), Appendix I, for the arch with symmetrical loads. For a more exact computation the neglected terms in Equations (75) and (76) may be obtained as shown by Professor Rathbun.[15]

If the abutments of a skew arch are likely to move due to the initial stresses, the error in computing the reactions depends upon the assumptions of the abutment displacements in six co-ordinates. The stress analysis may be simplified by introducing hydraulic jacks at the crown during construction, as shown in Fig. 38. Some of the unknown stresses at the crown may

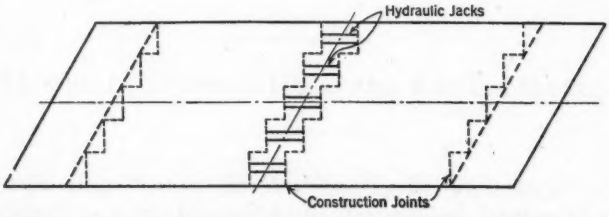


FIG. 38.—PLAN SHOWING HYDRAULIC JACKS METHOD OF POURING SKEW ARCH

be eliminated by properly detailing the jack bearings. Moreover, the arch may be pre-stressed to almost any desirable condition by assuming a temporary load.

It should be noted that a considerable departure from the fundamental requirements of the elastic theory of the statically indeterminate structures is made in applying the skew arch equations to the "rigid frame." This is due to the nearly right-angle corners in the neutral axis of the frame shown in Fig. 29.

For a rigid scientific analysis of stress distribution in a skew arch it has not yet been sufficiently proved that the effect of shear stresses at the sharp corner of a section, Fig. 12(b), will be carried uniformly over the entire width. Bearing in mind the elastic properties of concrete, it is more likely that the angle, θ (Fig. 14), decreases rapidly from the face toward the middle of the section.

The design of construction joints has tremendous influence on stress distribution in a skew arch. Experimental tests[16] with a hard rubber model of a skew arch has shown a symmetrical distribution of stresses due to the horizontal thrust parallel to spandrel walls and to the vertical reactions. This fact may be utilized in designing saw-toothed construction joints normal to the neutral axis in the arch and abutments, as shown in Fig. 38.

Numerous assumptions were made in these equations for stresses in a skew arch in order to simplify them and, therefore, experimental tests with models of a fairly large scale should be made to determine the magnitude of the errors introduced.

[16] *Engineering News-Record*, July 21, 1927, p. 106.

The author has performed a laborious task in presenting this contribution to the theory of skew arch design, and further tests are needed before conclusive statements can be made.

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DISCUSSIONS

A DISCHARGE DIAGRAM FOR UNIFORM FLOW IN OPEN CHANNELS

Discussion

BY MESSRS. L. STANDISH HALL, AND RAY E. MACKENZIE

L. STANDISH HALL,[11] ASSOC. M. AM. SOC. C. E. (by letter).[11a]—This paper offers a solution to a troublesome hydraulic problem frequently met in the measurement of stream discharge.

The influence of a water surface with variable slope on the discharge relation affects the results on streams more frequently than is ordinarily appreciated. When the change in slope is quite marked this condition is readily identified. However, in many instances, the effect of the variation in slope is so slight that it is confused with a shift of control. In cases where a variable surface slope is combined with a shifting control, the separation of the two effects is often quite difficult.

A control is effected on a stream, when the water surface proceeding in the direction of flow changes from a flatter to a steeper slope. Some controls are so slight that they are "drowned out" when higher stages are reached, and the condition of variable surface slope is introduced.

Before classing a station as one affected by a variable surface slope, a careful check on the condition of the intake pipe to the recorder well should be made, since a partly plugged intake pipe can create the effect of larger discharges on rising stages than on falling stages.

In Fig. 10 is shown the discharge curve for a part of the measurements taken on the Mokelumne River, at Woodbridge, Calif., for the year 1927. The separation of the curves for rising and falling stages is easy to distinguish. The interpretation of measurements at this station is further complicated by shifts in the control. The points plotted on the graph have been adjusted to represent times when the condition of the control was particularly uniform. The variation between the two curves does not exceed 10% of the flow, so

NOTE.—The paper by Murray Blanchard, M. Am. Soc. C. E., was published in January, 1931, *Proceedings*. Discussion on the paper has appeared in *Proceedings* as follows: May, 1931, by Messrs. Lynn Perry, J. B. McPhail, G. B. Pillsbury, Lynn Crandall, and Benjamin E. Jones; and September, 1931, by A. J. Matheson, Esq.

[11] Chf. Hydrographer, East Bay Municipal Utility Dist., Oakland, Calif.

[11a] Received by the Secretary August 14, 1931.

that, in this case, reasonably accurate discharges could be determined by the use of one average curve to represent rising stages and a second for falling and level stages. Although it is true that a transition from one slope to

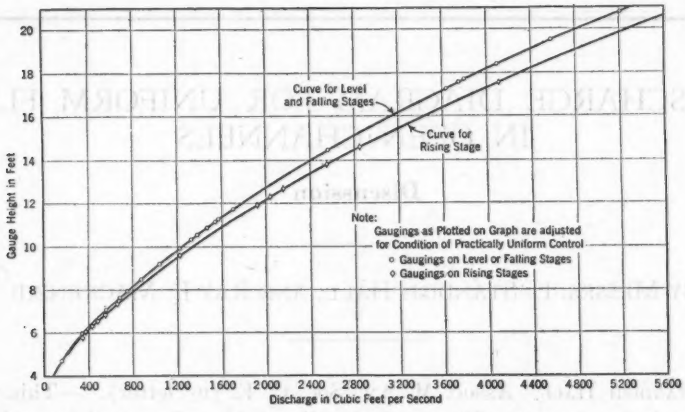

FIG. 10.—RATING CURVES, MOKELUMNE RIVER AT WOODBRIDGE FOR YEAR 1927, SHOWING EFFECT OF VARIABLE SURFACE SLOPE

another must exist, the results of the discharge measurements would appear to indicate that such transition periods are of brief duration.

RAY E. MACKENZIE,[12] ASSOC. M. AM. SOC. C. E. (by letter).[12a]—The author has afforded a good opportunity for discussion on the subject of back-water effect at a measuring station. While he has simplified the problem of determining the discharge at a station subject to back-water, further simplification is desirable. The form taken should be that which best suits the immediate purpose. If this is the reduction of gauge heights to daily discharge figures, there can be no better arrangement than the table. This must result from a graph, as all rating tables do, and will consume considerable time in preparation. However, it must be remembered that the task of reduction to discharge is the main one, and it is here that simplicity, speed, accuracy, and facility for checking are of paramount importance. The rating table is the common and accepted means of reduction, and, therefore, a simple form of it should be the more logical method of transfer.

The use of separate tables, one for the "normal" discharge and the other giving values of $\sqrt{z}$ would produce the desired result from a single multiplication, but two interpolations might be necessary. A tabular representation of Fig. 7, with columns headed "Fall, in feet," and lines to read "Elevation below Chicago City Datum," would condense the data into a single table. With attention given to the probable accuracy of the data, the table could be made so that hasty interpolation could be accomplished to the percentage of accuracy required.

[12] Hydro-Elec. Engr., Upper Mississippi Val. Div. Office, U. S. Engr. Dept., St. Louis, Mo.

[12a] Received by the Secretary August 28, 1931.

Numerous applications have been made of the principles involved in the paper. For some time the writer has used the principle in the reverse condition, that is, for back-water computation. The problem is the familiar one with back-water above a dam. The discharge to consider is known and the solution is for the back-water elevations at various points up stream.

Natural flood elevations and discharges are determined from field surveys with attention paid to the elevation and discharge at the time of the survey and to the probable point of zero flow. Often a satisfactory rating curve can be made and extended from as few as three points on full logarithmic paper.

A study of sections that form major and minor controls for various stages of flow will materially help the drawing of the curves and their extension to the elevations of probable back-water. Separate rating curves are drawn for each section forming an end of a reach. While these should be made so as to show properly the stage of zero flow (or approximately), it is very helpful in studying the curves and their extensions to draw the rating curves on a common scale of elevation. On such a graph the intersection of curves will show a level slope between the sections represented.

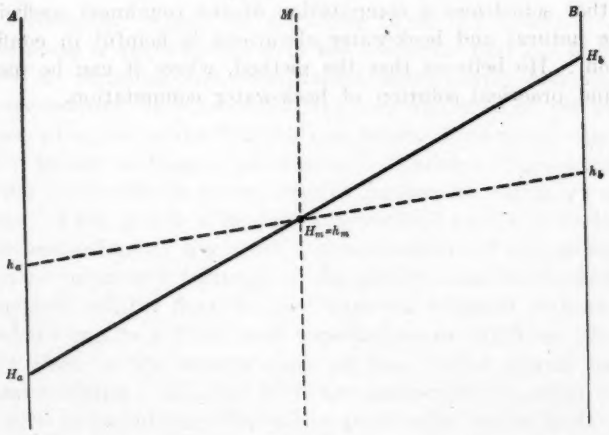

FIG. 11.

In Fig. 11, let F from Equation (2) equal $H_b - H_a$; in Equation (3) let $F_x = h_b - h_a$, and $q = Q_x$. Then, if Equation (12),

$$Q = K\,(H_b - H_a)^{0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

is used as the formula for natural flow, and Equation (13),

$$q = k\,(h_b - h_a)^{0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

for back-water flow, and, furthermore, if h_m is made equal to H_m, then k equals K, and the author's Equation (5) becomes,

$$h_b - h_a = (H_b - H_a)\,\frac{q^2}{Q^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

In back-water computations, q is known and the point, h_a, is determined from the computed rise or back-water at the dam or obstruction. Then, if the assumption can be made that $h_m = H_m$, a working basis will be found to proceed with the back-water computation. The order follows:

(1) Assume a value of h_m that equals H_m.
(2) Enter the rating curve for Section M with H_m and read Q.
(3) With Q known, H_b can be read from the rating curve for Section B, and H_a can be read from the rating curve for Section A.
(4) In Equation (14) substitute values for q, Q, H_b, and h_a; then compute h_b (the new back-water point).

The method is, of course, a cut-and-try one, but it gives good approximate results when a poor assumption of $h_m = H_m$ is made. If refinement is required a possible second trial should be sufficient to produce a satisfactory computation of the back-water elevation.

Since the rating curve for Section M is the average of the B and A curves, its elevations will be midway between them, and the curve for M need not be drawn.

The writer has used the method and variations of it for several years. He finds that sometimes a computation of the roughness coefficients for a few of the natural and back-water elevations is helpful in confirming the computation. He believes that the method, where it can be used, enables a useful and practical solution of back-water computation.

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DISCUSSIONS

SURVEYS FOR SWIFT RIVER RESERVOIR OF THE BOSTON METROPOLITAN WATER SUPPLY

Discussion

BY FRANK C. SELLNOW, ASSOC. M. AM. SOC. C. E.

FRANK C. SELLNOW,[6] ASSOC. M. AM. SOC. C. E. (by letter).[6a]—Although technical literature is quite replete with the procedure that has been successfully followed in planning and constructing water supply and other major hydraulic undertakings, little seems to have been published on the methods of surveying, mapping, and acquiring the real estate of reservoir sites as these methods have tended to become standardized. Perhaps it has been thought that surveying is too old an art to require further exposition as applied to reservoir sites. That this is a mistake is evidenced by the fact that all too often in the past—through too great a subordination of this work to other seemingly more important features of the project—no well-considered program was mapped out, the data obtained were not collected with an economy of effort, and the results did not meet expectations or fulfill requirements.

Reservoir sites in the eastern part of the United States have many common characteristics. All, naturally, are surrounded by relief which will range from hills to mountains; the valleys are usually fertile land, cultivated, and of fairly high value, and there will be some utilities, both public and private. Community life will usually center in one or more villages in which, also, are occasionally found industries requiring large quantities of water. The upland will usually be wooded and, compared with the bottom-lands, of low value. The area within the taking line will seldom be less than 5 sq. miles, or more than 100 sq. miles, depending upon the gradient of the stream and the size of the development. With this similarity in the physical features, standardization of surveying and mapping methods, with such slight modifications as are necessary to meet local conditions, would seem to be invited.

NOTE.—The paper by N. LeR. Hammond, Esq., was presented at the meeting of the Surveying and Mapping Division, Boston, Mass., October 10, 1929, and published in February, 1931, *Proceedings*. Discussion on the paper has appeared in *Proceedings* as follows: May, 1931, by Messrs. C. B. Breed and C. L. Hall.

[6] Div. Engr., State Water Policy Comm., Newark, N. J.

[6a] Received by the Secretary, June 2, 1931.

Due, perhaps, to the low average value of the land, maps, if any exist, are almost invariably of low-grade accuracy, or are published on too small a scale to be of value in the design of water-works structures, relocation of utilities, or the acquisition of the real estate. The topographical maps of the United States Geological Survey, supplemented by reconnaissance, will usually suffice to determine the feasibility of the project, but before detailed plans can be made a topographic map, preferably to a scale of not less than 1 in. equals 200 ft., must be available. Enlargements of the topographical maps of the United States Geological Survey will only have the effect of accentuating the inaccuracies inherent in drafting these or any small scale maps and will not produce a map which will fulfill requirements.

Simultaneously with the topographical survey, or at least well in advance of the flooding of the reservoir, a cadastral survey must be made for the purpose of acquiring all the real estate. Fortunately, the locus approaches the ideal for carrying out both these surveys with a high order of accuracy and, at the same time, at very reasonable costs. The manner in which the Swift River Reservoir surveys were conducted demonstrates an orderly and efficient procedure for collecting the necessary data. It would be interesting and useful information if the cost of the Swift River Reservoir survey and the surveys of other reservoirs—classified, say, into triangulation, traverse, real estate, and topography—were published. On a unit of area basis, there should be no great variance in cost.

If the terrain is not too heavily wooded, it can be stated confidently that the control for both surveys can be established more economically by triangulation than by precise traverse. Considerable thought and study will have to be devoted to locating the triangulation stations to obtain intervisibility, to avoid excessive cutting, and to keep within a 30° angular limitation. In the reconnaissance, the use of an aerial map should be a very useful adjunct. To avoid the displacement or destruction of stations by utility relocations or construction activities, they should be located at least 50 ft. higher in elevation than the future flow line. They should be preserved by monumentation and properly referenced, so that their usefulness will be continued should it be found desirable in later years to increase the real estate holdings or to make improvements. The surest reference for the control system is to tie, by traverse or triangulation, the origin of the local system of plane co-ordinates to one or more of the control stations on the North American datum. This will have the advantage not only of perpetuating the survey, but of eliminating all astronomical observations, and making it possible to orient and locate properly the survey data on any Government or State maps.

In establishing the triangulation control for the Swift River Reservoir survey, the author was fortunate in that the township corners were marked and their geographical positions known. These geographical positions, transposed into rectangular co-ordinates, furnished, as stated in the paper, several checks to the new system. This condition of frequently located monuments, the geographical co-ordinates of which are known, will not ordinarily obtain in most of the Eastern States, and without these checks a greater refine-

ment in the measurement of the angles would be required to secure the same accuracy. Two sets of pointings—each consisting of six measures of the angle and six measures of the explement, with an instrument graduated to 20″—would be the least number of angular measurements deemed necessary for this class of work.

Although the several steps in establishing the control and in making the cadastral and topographical surveys of reservoir sites may be performed in much the same way, the degree of required accuracy will depend upon valuations as between two reservoir sites or between different parts of the same reservoir. In general, the higher the valuations the greater the accuracy demanded. In any event, the standard adopted for the cadastral work will meet all the requirements of the topographical map. It is believed that the following specifications for accuracy for maps plotted to a scale not greater than 200 ft. to the inch, will meet the most exacting requirements, whether distances are to be mathematically reduced or scaled: (*a*) Triangulation, 1 part in 20 000; (*b*) traverses, the stations of which are to be used for locating property corners, or as the starting and closing points of stadia surveys, 1 part in 10 000; and (*c*) horizontal distances between well-defined points as located by stadia, to scale accurately within 3 ft.

Engineers, for the most part, now recognize that aerial photography will aid and supplement the ground survey of any large tracts. The author made commendable use of this comparatively new art, but it is questionable whether he availed himself of its full possibilities. The aerial survey made was uncontrolled. Control could have been furnished by the valuation maps of the Boston and Albany Railroad Company, providing they contain the required data; if not, then a moderate amount of ground control would have been necessary, the expense of which, it seems safe to state, would have been more than offset by the benefits. It is believed that controlled mosaics to a scale of 200 ft. to the inch, on sheets to correspond to the co-ordinate layout, could have been obtained at not more than four times the cost of the contact prints. Mosaics in this form would have obviated pantographing and furnished, by direct comparison, a check on all property surveys. As most aerial survey companies will guarantee an accuracy within 0.25% in distances of 5 000 ft., or more, topographical features within areas surrounded by natural monuments (such as river, roads, etc., located by ground method), could have been transferred to the line map with the assurance that their positions would have been at least within the limitations of the stadia method. Contours would have to be obtained by the usual field methods.

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DISCUSSIONS

FLOW OF GROUND-WATER AS APPLIED TO DRAINAGE WELLS

Discussion

BY O. W. ISRAELSEN, M. AM. SOC. C. E.

O. W. ISRAELSEN,[10] M. AM. SOC. C. E. (by letter).[10a]—The study of flow of ground-water toward wells is not only of scientific interest, but also of real practical importance. Mr. Lewis is to be congratulated on the thorough and valuable analyses presented.

The term, transmission "constant," was used by Slichter, not transmission "coefficient." In defining the transmission constant or coefficient, care should be exercised to specify unit pressure head difference. That part of the author's definition ("Type of Wells") of the transmission coefficient which reads, "under the pressure of a column of water of unit height," is clearly incomplete.

In a paper entitled "The Drainage of Land Overlying Artesian Basins," [11] by Messrs. Willard Gardner, W. W. McLaughlin, and the writer, an equation analogous to the author's Equation (3) was derived, namely:

$$h = \frac{Q_2}{\rho} \times \frac{\log_e \frac{r}{R_1}}{2\pi\, Dg\, k_2} \qquad (23)$$

in which, in addition to the author's notation:

Q_2 = mass of water per unit time.

ρ = density.

g = acceleration due to gravity.

$k_2 = \frac{k}{g}$

NOTE.—The paper by M. R. Lewis, M. Am. Soc. C. E., was published in March, 1931, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: In September, 1931, by L. Standish Hall, Assoc. M. Am. Soc. C. E.

[10] Prof., Irrig. and Drainage Eng., Utah State Agri. Coll., Logan, Utah.

[10a] Received by the Secretary August 31, 1931.

[11] *Soil Science*, Vol. 26, No. 1 (1928).

The author's k has the physical dimensions of $\frac{L}{T}$. The symbol, k_2, is a transmission constant that has the physical dimensions of time; thus,

$$\frac{k_2}{g} = \frac{\frac{L}{T}}{\frac{L}{T^2}} = T.$$

In commenting on Equation (5) the author states that: "It will be noted that the factor, w, is somewhat analogous to the transmission coefficient, k." This statement could well be made more direct. As shown in Equation (7), w is proportional to the transmission coefficient of the overlying, relatively impervious material, k_1. The quantities, w, k, and k_1, are dimensionally the same.

Field studies of the cone of depression in the piezometric surface near the axis of the pump in an experimental drainage well in Utah, of the Type Ab, confirm in general the requirements of Equation (5).[12]

The author's Equations (10) and (21) and their applications are of special interest. It is important to note that for Type Ab wells "satisfactory drainage" at a given distance from a well is not only a function of h, but also of the transmission constant, k_1. Equation (8) shows that as k_1 increases, m decreases, for any given values of h, w, and T. Satisfactory drainage with the Type b wells is also dependent on the quantity of irrigation water applied, which influences the quantity, w.

Measurements of the piezometric surface, as influenced by pumping from a Type Ab well, in Utah, show significant reductions of pressure head at distances considerably more than 1 000 ft. In this case, it would seem that if k_1 were fairly large and m relatively small, satisfactory drainage may be secured at a distance from the well several times greater than 300 ft.

All the equations derived by the author, together with the diagrams illustrating their applications to typical cases, are of genuine interest to engineers who are concerned with pumping water from ground-water sources whether for drainage or for irrigation purposes.

The increasing "steepness" of the cone of depression of the piezometric surface as the axis of the well is approached, especially for water-bearing materials having a relatively low transmission coefficient, stresses the importance of research concerning design of wells with a view to flattening the cone economically and reducing the power requirements by decreasing the heavy draw-down.

[12] *Proceedings*, Utah Academy of Sciences, Vol. 8, p. 35 (1931).

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DISCUSSIONS

AN ANALYSIS OF MULTIPLE-SKEW ARCHES ON ELASTIC PIERS

Discussion

BY MESSRS. CHARLES L. NORD, F. W. KEYSER, AND A. A. EREMIN

CHARLES L. NORD,[26] M. AM. SOC. C. E. (by letter).[26a]—To any one familiar with the first paper[27] written by Professor Rathbun, it will be perfectly apparent that the method presented in this paper is, in the main, the same as that used in multiple right arches, in which the reaction of one is considered in the computation of the other.

The arch rings are divided into cantilevers, loaded with uniform and concentrated loads and with the reaction of the cutting plane or planes as in the case of the intermediate spans. The entire mathematical analysis of the single skew arch as well as the multiple arches is based on the flexure theory and can readily be understood and applied by engineers familiar with the elastic theory.

Other methods, such as "virtual work," or "least work," could be used. These, however, are too tedious and complicated for the average designer.

Professor Rathbun's mathematical analysis of the single skew arch is of primary importance to the profession since it has been proved by tests that, for vertical loads, the reactions are transferred to the supports along the lines parallel to the skew axis and not along those normal to the support. This, apparently, is contrary to the fundamental law of Nature that work is done by forces along the shortest lines and in the most economical way; although the writer fails to see where there can be an error in Professor Rathbun's method.

That other authorities do not agree with the author may be seen from a paper[28] by Professor Oskar Muy, of the Technische Hochschule, Stuttgart, Germany, describing the design and construction of a reinforced skew arch bridge over the Saar River, near Voelklingen. In this paper it is stated

NOTE.—The paper by J. Charles Rathbun, M. Am. Soc. C. E., was published in April, 1931, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1931, by Messrs. B. L. Weiner and G. D. Houtman.

[26] Bridge Engr., State Highway Dept., New Haven, Conn.

[26a] Received by the Secretary, June 29, 1931.

[27] "Analysis of Stresses in the Ring of a Skew Arch," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

[28] *Deutsche Bauzeitung*, Nos. 14–18, 1915.

that in a skew arch the stresses cannot follow the arch as far as the sharp angles at the haunches, but will run more in the direction of the plane normal to the abutments, and that, therefore, the tendency is to overstress the obtuse corners greatly, with the likelihood of breaking them. To meet this situation, these points were heavily reinforced in a transverse direction. Professor Muy advanced no theory to support his view, but based his assumption solely on what he called "common sense." This would seem to indicate that a theory or method of designing a skew arch was a universal need of the Engineering Profession.

TABLE 6.—COMPUTATIONS FOR AN 80-FOOT SPAN OF A SKEW ARCH.

Term	Value	Term	Value
$\Sigma \cos^2\phi \frac{s}{A}$	0.47*	$\epsilon\,\lambda \Sigma x \sin\phi \cos\phi \frac{s}{F}$	0.12
$\lambda \Sigma \sin^2\phi \frac{s}{A}$	0.32*	$\Sigma z^2 \frac{s}{I_y}$	1.20*
$\Sigma y^2 \frac{s}{I_z}$	32.37	$\lambda \Sigma u^2 \frac{s}{F}$	187.40
$e^2 \Sigma x^2 \cos^2\phi \frac{s}{I_y}$	0.47*	$\lambda \Sigma \frac{s}{A}$	1.51*
$e^2 \lambda \Sigma x^2 \sin^2\phi \frac{s}{F}$	58.14	$\Sigma v \sin\phi \frac{s}{I_y}$	0.017*
$2h \Sigma y \frac{s}{I_z}$	95.77	$\lambda \Sigma u \cos\phi \frac{s}{F}$	0.13
$h^2 \Sigma \frac{s}{I_z}$	569.38	$\Sigma \sin^2\phi \frac{s}{I_y}$	0.0001*
$\epsilon \Sigma VX \cos\phi \frac{s}{I_y}$	0.79*	$\lambda \Sigma \cos^2\phi \frac{s}{F}$	1.295
$\epsilon\,\lambda \Sigma u x \sin\phi \frac{s}{F}$	105.00	$\Sigma \cos^2\phi \frac{s}{I_y}$	0.004*
$\epsilon \Sigma x \sin\phi \cos\phi \frac{s}{I_y}$	0.008*	$\lambda \Sigma \sin^2\phi \frac{s}{F}$	0.137

* Terms in Equation (11) that may be eliminated.

The writer unintentionally checked Professor Rathbun's statement that certain terms may be eliminated and found it correct. Table 6 was prepared in the computation of a skew arch of 80 ft. span, at an angle of 30°, which, later, was checked by Professor Rathbun personally.

F. W. KEYSER,[29] ESQ. (by letter).[29a]—The importance of Professor Rathbun's mathematical achievement is obvious when one realizes the many grade separations made necessary by highways crossing other highways, railroads, and rivers. The constantly expanding public roads program and its accompanying economy has made, for the skew arch, a definite place in the world of structures.

[29] New Haven, Conn.

[29a] Received by the Secretary August 24, 1931.

The mathematical analysis advanced by Professor Rathbun in this paper and in his previous paper on skew arches, namely, "Analysis of the Stresses in the Ring of a Concrete Skew Arch," [30] ends the long period of obscurity in which the skew-arch problem has rested.

The slow increase in bridge width has been greatly accelerated in recent years, and, in a very short time, the effect of eccentric loads on the right arch will be a primary and not a secondary factor in the design of right arches. This eccentricity of loads on right arches can no longer be neglected. Professor Rathbun's analysis with its accompanying fundamental assumptions definitely shows the effects of eccentric loads. Not only that, but it points the way to the solution of eccentric stresses in the design of arches in the future.

The fact that Professor Rathbun's analysis is an exact one and is based on simple fundamental assumptions does not eliminate the case in which the stresses, computed as such, would be only approximate at best. For example, he has assumed rigid supports which means absolutely unyielding foundations, since the reactions do no work. This condition is never quite encountered in practical cases because even rock has a certain degree of elasticity and will deform under the action of thrusts and moments (although the deformation is insignificant).

For most other soils the deformation will be greater and more permanent, and, therefore, the writer believes that it remains for the designer to decide how far to go in accepting soil conditions as found and, at the same time, consider the work of the reactions as zero. In other words, Professor Rathbun's analysis is based on an assumption that never actually occurs in practical cases, although it may be approached.

The deformations as calculated are comparatively small; they will have to be so, in order to apply the elastic theory. If a settlement of the foundation occurs, it can at best only be estimated; it certainly cannot be calculated. The calculated deformations and, therefore, the crown reactions which are dependent upon the rigidity of the support, will necessarily be approximate only. In regard to the author's assumption of rigid supports the writer would like to issue a solemn warning that the skew arch, as well as any other indeterminate structure, should not be considered unless absolutely reliable data on the subsoil condition are at hand, to be studied carefully and deliberately, so as to determine the exact degree of rigidity that can be relied upon. In other words, engineers are all too apt to equate the work of the abutment reactions to zero. In many instances this is not so, due to subsoil conditions that are not apparent until the excavation for the structure has started. Then, spread footings, tie-beams, pile foundations, and other methods are used to make up for the lack of assumed rigidity. In such cases the indeterminate structure would not be economical and might even prove dangerous.

The modulus of elasticity is another factor to be considered. The writer is fully aware of the fact that common practice of present-day design is to consider this factor a constant when dealing with materials that follow the constant ratio of stress to deformation. Within the limits of stress commonly

[30] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

used, this assumption is correct; that is, if concrete is one of these materials. Tests made by Professor Emil Probst, in the testing laboratory in Zurich, published in 1906, give a different picture. Dr. Ing. Rudolph Saliger[31] has called attention to several tests made by Professors Bach, Wayss, and Freytag, of Berlin, which indicate that concrete is not one of these materials. The results of these tests show that the modulus of elasticity for concrete is a function of the stress, and, therefore, must be treated as a variable and not as a constant. However, the striking of an average stress deformation constant might be justifiable for an approximation except for another complication that enters in concrete structures, namely, the initial stresses induced in the steel and concrete during the setting process of the concrete. This subject has received little attention and much less investigation and, therefore, must be considered unsolved. These stresses may be of such intensity as to cause considerable variation in the stress computed by the elastic theory.

Another subject that has received very little attention and investigation is the behavior of large sections under torsional moments. Their behavior is most uncertain and this problem must be considered as unsolved.

These points have been cited to show why structures of the skew-arch type should be designed with conservative stresses, the judgment of experience as to conditions of site, and an absolute understanding of the analysis and its assumptions. The writer's views are those of the practicing engineer and not of the scientific investigator and mathematical expert. They are given in the spirit of co-operation to the end that a practical solution of the problem may be accepted by the profession.

With these points in mind, and under the assumption that they may be discounted, the writer has made a study of Professor Rathbun's analysis of skew arches by the Method of Virtual Work, or as it is more commonly known, the Principle of Least Work. He has checked his formulas for the single-span skew arch as outlined in Part I of the paper, but in the development of Equations (3) for the right side, sign differences were encountered because the directions of forces and moments were assumed to act as in Professor Rathbun's first paper,[32] where they are opposite in direction to that in the present paper. These sign differences occur in the writer's analysis only when based on his own interpretation of the Method of Virtual Work and the sign system for the right half of Arch *A* (Fig. 3) as developed in Professor Rathbun's first paper.[32] By assuming that the directions of moments, forces, and crown deformations, as shown in Figs. 6 and 7, are correct, the writer has checked Professor Rathbun's analysis in sign and terms for both the right side and the left side.

The method of virtual work is based upon Castigliano's first theorem:[33]

"The derivative of the total internal work, W, with respect to any one load or external force, X, is equal to the displacement, Δ, in the direction of this load."

[31] "Ueber die Festigkeit Veränderlich Elasticher Konstructionen," Leipzig, 1904.

[32] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 613.

[33] "Die Neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen," and, "Die Graphische Statik der Baukonstruktionen," by Prof. H. Müller-Breslau. (Lectures of the same Professor as recorded by the writer during his student days.)

In other words,

$$\frac{dW}{dX} = \Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

The general formula for the elastic work is,

$$W = \tfrac{1}{2}\int \frac{t_v^2\,ds}{EA} + \tfrac{1}{2}\int \frac{m_z^2\,ds}{EI} + \tfrac{1}{2}\int \frac{t_u^2\,ds}{GA} + \tfrac{1}{2}\int \frac{m_u^2\,ds}{GI_p} \quad \ldots\ldots (44)$$

in which, G = modulus of shear $\left(= \dfrac{E}{\lambda}\right.$ in Professor Rathbun's analysis); and,

GI_p = factor of torsion $\left(= \dfrac{EF}{\lambda}\right.$ in Professor Rathbun's analysis).

Differentiation of Equation (44) by partial differentiation for the unknown elastic values, which is denoted by X, determines the virtual work; thus,

$$\frac{dW}{dx} = \Delta = \int \frac{t_v}{EA}\frac{\partial t_v}{\partial x}\,ds + \frac{m_z}{EI}\frac{\partial m_z}{\partial x}\,ds + \lambda\int \frac{t_u}{EA}\frac{\partial t_u}{\partial X}\,ds + \lambda\int \frac{m_v}{EF}\frac{\partial m_v}{\partial x}\,ds. \quad (45)$$

When the abutment deformation is neglected Equation (45) represents $1 \times$ Deformation. The modulus of elasticity, E, considered constant, is next removed from the integrals and multiplied by the deformation.

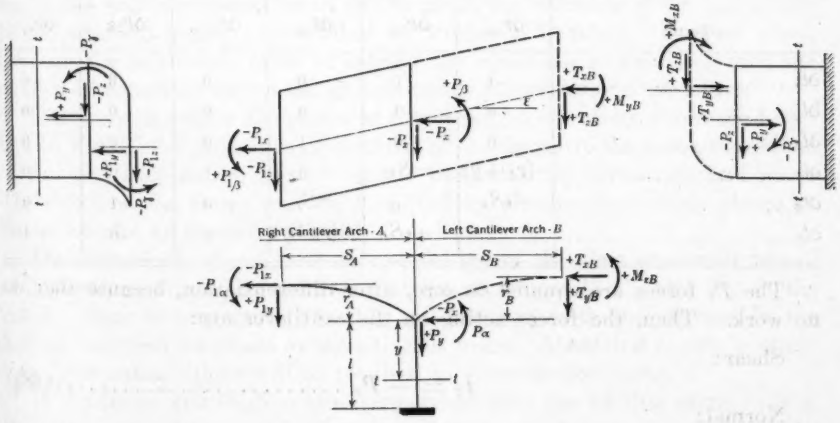

FIG. 29.

In his analysis the writer treated the problem as a single cantilever, the right cantilever of Arch A, acted upon by the forces of Arch B, as shown in Fig. 29. The P forces acting on the cantilever are due to the crown reactions of Arch B and their relationship may be demonstrated as follows:

$$P_x = T_{xB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

$$P_y = T_{yB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46b)$$

$$P_z = T_{zB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46c)$$

$$P_\alpha = T_{xB}\, r_B - T_{yB}\, S_B + M_{zB} \quad \ldots\ldots\ldots\ldots (46d)$$

$$P_\beta = T_{xB}\, \epsilon\, S_B - T_{zB}\, S_B + M_{yB} \quad \ldots\ldots\ldots\ldots (46e)$$

$$P_\gamma = T_{yB}\, \epsilon\, S_B - T_{zB}\, r_B + M_{xB} \quad \ldots\ldots\ldots\ldots (46f)$$

The P_1 forces are the unit forces applied in the direction of the deformations caused by the P forces. The relationship of the forces acting on the Section (t-t) of the pier is shown by the following formulas:

$$t_x = -P_x - P_{1x} \ldots\ldots\ldots\ldots (47a)$$

$$t_y = +P_y + P_{1y} \ldots\ldots\ldots\ldots (47b)$$

$$t_z = -P_z - P_{1z} \ldots\ldots\ldots\ldots (47c)$$

$$t\alpha = -P\alpha - P_1\alpha - P_x y - P_{1x}(r_A + y) - P_{1y} S_A \ldots\ldots (47d)$$

$$t_\beta = -P_\beta - P_{1\beta} + P_{1x}\, \epsilon\, S_A - P_{1z} S_A \ldots\ldots\ldots\ldots (47e)$$

$$t_\gamma = -P_\gamma - P_{1\gamma} - P_{1y}\, \epsilon\, S_A - P_{1z}(r_A + y) + P_z y \ldots\ldots (47f)$$

The partial differentiation of Equations (47) with respect to the P_1 forces are given in Table 7.

TABLE 7.—PARTIAL DERIVATIVES FOR USE IN EQUATIONS (47)

Terms	∂P_{1x}	∂P_{1y}	∂P_{1z}	$\partial P_{1\alpha}$	$\partial P_{1\beta}$	$\partial P_{1\gamma}$
∂t_x	-1	0	0	0	0	0
∂t_y	0	$+1$	0	0	0	0
∂t_z	0	0	-1	0	0	0
∂t_α	$-(r_R+y)$	$-S_R$	0	-1	0	0
∂t_β	$-\epsilon S_R$	0	$+S_R$	0	$+1$	0
∂t_γ		$+\epsilon S_R$	$+(r_R+y)$	0	0	-1

The P_1 forces are equated to zero, after differentiation, because they do no work. Then, the forces acting on the cantilever are:

Shear:

$$t_x = -P_x \ldots\ldots\ldots\ldots (48a)$$

Normal:

$$t_y = +P_y \ldots\ldots\ldots\ldots (48b)$$

Shear:

$$t_z = -P_z \ldots\ldots\ldots\ldots (48c)$$

Moment:

$$t_\alpha = -P_\alpha - P_x y \ldots\ldots\ldots\ldots (48d)$$

Torsion:

$$t_\beta = +P_\beta \ldots\ldots\ldots\ldots (48e)$$

Moment:

$$t_\gamma = -P_\gamma + P_z y \ldots\ldots\ldots\ldots (48f)$$

By inserting the values of the shears, normal force, moments, and torsion, as obtained from Equations (48), with their respective differential

values, in the general formula for virtual work (Equation (44)), the equations of deformations in terms of the P forces are determined. By the proper substitution of the crown forces of Arch B (Equations (46), in the equations of deformation, the author's Equations (3) are derived except that, as noted previously, the signs are reversed completely in Equations (3*a*), (3*c*), (3*d*), and (3*f*).

It will be noted that the load terms have been neglected. The writer considered the cantilever unloaded. The load terms are independent of the elastic values and should be treated separately. Their omission will not affect, in any way, the equations as developed.

In closing, the writer wishes to acknowledge the assistance of Joseph D. Drury, a co-worker in the Engineering Division of the Connecticut State Highway Department, in checking the mathematical analysis and in the preparation of this discussion.

A. A. EREMIN,[34] ASSOC. M. AM. SOC. C. E. (by letter).[34a]—The author has extended the application of the deflection equations of a cantilever skew arch to the solution of a continuous multiple-span rigid frame.

The equations in the terms of the loading and their reactions at the cut planes developed by the author may be solved either simultaneously for all spans of the frame, or in the groups of six equations for each span, by substituting the unknown reactions in the terms of the reactions at the cut planes in the adjacent spans, as described by Professor Rathbun. In both cases, however, the arithmetic error in solving the equations accumulates with the number of equations connected by continuous properties of a structure.

If a frame is cut by the planes at the crown, as shown by Professor Rathbun in Fig. 3, the procedure of solution of the frame is the same, regardless of the variation in the relative elastic properties of the arches and piers. The solution of a frame may be simplified by placing the cutting planes at the skewbacks, as shown in Fig. 30.

The deductions, Δ_Y, Δ_X, and $\Delta\gamma$, at the top of the piers are usually small and, therefore, it may be assumed that the reactions T_y, T_z, and M_x, at the cutting plane of one span (Fig. 30) will not vary with the changes in the loading and their reactions at the adjacent spans. According to this assumption fewer substitutions will be required to solve the equations.

If the frame and loading are symmetrical then one cutting plane in Fig. 30 may be placed symmetrically to the frame and the other sections at the skewbacks in the adjacent spans.

It is interesting to compare Equation (8), used by Professor Rathbun for computing the unit shear stress due to torsion moment, to Equation (49) which was experimentally developed by Professor Bach[35]:

$$v = \left(3 + \frac{2.6}{0.45 + \frac{b}{d}}\right)\frac{f}{bd^2} \qquad \text{.....(49)}$$

[34] Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.

[34a] Received by the Secretary October 13, 1931.

[35] "Deutscher Ausschuss für Eisenbeton," Heft 16, 1912, W. Ernst & Sohn, Berlin.

The numerical coefficients in both Equations (8) and (49) will be the same for the cross-section with the ratio, $\frac{b}{d} = 1.29$.

In his test, Professor Bach used beams of a square cross-section and the rectangular section with the ratio, $\frac{b}{d} = 2$. Therefore, his equation and, apparently, Equation (8) cited by Professor Rathbun, are true only in this range of the sections.

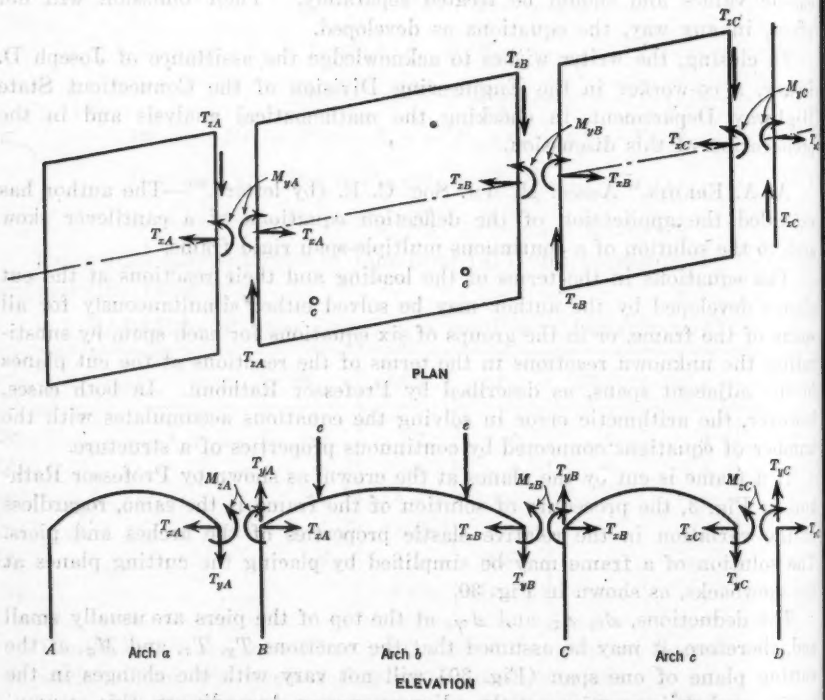

FIG. 30.—MULTIPLE-SPAN RIGID FRAME DIVIDED BY CUTTING PLANES.

The writer agrees with Professor Rathbun that the shear reinforcing steel should be placed at right angles to the main reinforcement. A very noticeable loss in the efficiency of steel reinforcement occurs if it is placed inclined to a force. This may be proved by simple geometrical relations.

In Fig. 31 is shown a steel bar, AB, inclined to the force, P, at an angle, α. If the length of a strand, l, changes to $l + \delta$, the strain in the bar will be:

$$\frac{\delta}{l} \cos^2 \alpha \quad \ldots\ldots (50)$$

Whence, the stress in the bar in the direction of the force, P, is:

$$E \frac{\delta}{l} a_z \cos^3 \alpha \quad \ldots\ldots (51)$$

Therefore, the effective area of the cross-section of the inclined bar is:

$$a_s \cos^3 \alpha \quad \ldots\ldots\ldots\

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DISCUSSIONS

RUN-OFF—RATIONAL RUN-OFF FORMULAS

Discussion

BY MESSRS. REGINALD A. RYVES, G. S. TAPLEY, W. I. HICKS, CARL H. REEVES, JOHN M. KEMMERER, LEONARD L. LONGAGRE, G. H. HICKOX, AND DONALD M. BAKER

REGINALD A. RYVES,[20] ESQ. (by letter).[20a]—This paper represents a formidable mathematical effort which prompts the question, "Why seek a formula when all data and assumptions can be applied directly by a graphical statement?"

The first necessity in making calculations of the flood discharges of catchments is to have some means of comparing any catchment with any other catchment, even when they differ much in size. Comparisons by means of rates of flow per square mile are of little use, since an accompanying scale of the effect of size has to be used in order that the comparison may produce a relation which represents rainfall and run-off conditions only. On the other hand, formulas involving a consideration of the shape of the area are not satisfactory, because, in comparing two catchments, more than one variable is involved, so that a relation which represents the rainfall and run-off conditions only is not arrived at, although the formula involving shape may give a more accurate result in the estimate of the run-off of a particular catchment.

If shape is to be considered, orientation is, in some cases, of great significance.

The only satisfactory way of making general comparisons is the use of a simple formula, such as that of Colonel Dickens, first applied in Bengal:

$$Q = C_1 M^{\frac{3}{4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

or that of Maj. J. G. Ryves, first applied in Madras:

$$Q = C_1 M^{\frac{4}{5}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

In Equations (76) and (77) C_1 ranges normally from 450 for flat districts near the coast to 700 for limited areas near the hills. However, in the great floods that occur about twice in a century, C_1 will equal 1 000 in many areas. Both equations have been in frequent use in India for more than fifty years, and the choice of one or the other definitely depends on topography, the former

NOTE.—The paper by R. L. Gregory and C. E. Arnold, Associate Members, Am. Soc. C. E., was published in April, 1931, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1931, by Messrs. Le Roy K. Sherman, Francis Bates, and John W. Raymond, Jr.

[20] Cons. Engr., London, England.

[20a] Received by the Secretary July 29, 1931.

in relatively infrequent conditions, while the latter is of almost world-wide applicability. By Equation (77) the flood run-offs of a large number of catchments in the arid and semi-arid areas of the United States, as well as those of catchments in India and other countries, have been made available as bases of comparison, and as checks on the general correctness of detailed calculations. The coefficient is the criterion.

The likely coefficient for a given catchment is found by considering the coefficients in a number of cases in which the area is not very small or very large in comparison, and where the intensity of a short-period rainfall is about the same.

More significant data are provided when: (*a*) The topography is somewhat similar; (*b*) the area is not more than three times nor less than one-third the area under consideration; and (*c*) the intensity of rainfall for the significant period is about the same. The significant period is the period of total contribution, or the time it takes for water to flow from the part of the catchment most distant, in time, to the outlet. In some cases, however, a smaller area, eliminating salients, gives a larger run-off because of the greater rainfall intensity.

Such comparisons are very useful and important; but for final estimates of floods the writer always uses the most careful methods of direct calculation, by methods of his own,[21] which are quite probably also followed independently by other engineers, and may be held to represent the rational method as criticized by the authors. The procedure is as follows, the specific references being to the writer's computations made for the "Obras Contra as Seccas" Department of the Government of Brazil, in 1920-1922:

1.—Rainfall maps, made in exactly the same manner as those of annual rainfalls, are prepared, except that one is made for each day, for the appropriate period; that is, a period long enough to overlap the "period of total contribution" by about 50% before, and 50% after.

2.—The area is divided into a suitable number of portions, usually four or five in the cases under consideration, and these portions are either each a catchment in itself, approximately, or they are zones, the bounding lines of which will divide the streams which have a 4 hours' run to the outlet, from those which have an 8 hours' run to the outlet, etc. There will be greater differences for larger catchments. The choice between these two methods of forming elements of area is decided by the shape of the catchment and the direction in which the streams run.

3.—From the part of the rainfall map, of each day, for each element of the area the daily rainfall is calculated, zone by zone, and the average found, for the purpose of a general check.

4.—From curves of daily run-off, the run-off for each element of area is calculated.

5.—The velocities of different lengths of the main stream and its affluents are estimated, from any available data, or by judgment, and the period of delay is deduced for each element of area, or zone. This is the period that

[21] *Indian Engineering*, April 29, March 6, and May 11, 1911.

must elapse before the water flowing from the element will reach the outlet. It is measured from the center of the area, or, if all the areas are of about the same size, and are roughly each a catchment, it may be measured from the outlet of the area. In the latter case the "gathering time" for an element is computed, and this is a "general handicap" for all the run-off curves as plotted to the time base-line.

6.—On a base line, marked off in hours to an open scale, the daily run-offs are plotted, first as horizontal lines, and then as continuous curves, bounding the same area as the horizontal line, each day; but all these curves do not start at the same point. Each is given a handicap, to the left of zero time, this handicap being, in each case, the period of delay. The longest handicap, plus the "gathering time" of that element, is the "period of total contribution."

7.—By totaling the heights of the run-off curves which are in cubic feet per second, at every hour-point for very small catchments or at longer intervals for larger catchments, the curve of total run-off is obtained.

The maximum run-off and the time of the maximum run-off are obtained from Steps 1 to 7 directly. The engineer is now able to judge, from the position of the time of maximum run-off on the base line, whether he is likely to get a greater run-off by taking a shorter period with a greater intensity of rainfall; or by taking a longer period with a less intensity of rainfall. Usually, however, all that he needs to do is to take a period that includes the day of greatest recorded rainfall, which in the case of the larger catchments, may involve the preparation of more than one map, since different stations may not have recorded their maxima in the same storm.

As the rainfall stations in Northeast Brazil are not very close together, the writer assumed that the real maximum spot rainfall is heavier than that recorded at any station, which is a matter of importance in estimates of floods from the smaller catchments

As regards estimating the velocities of the streams, this might seem to be a weak link in the chain of reasoning, but the records of the Department, which include curves of rainfall and run-off plotted together, make it possible to find, in a number of cases, not only the time elapsing between any great intensity of rainfall and the corresponding intensity of run-off, but also the period elapsing between the beginning of a heavy rainfall on the dry catchment, and the moment when the stream at the outlet begins to flow.

When the "period of total contribution" has been found, or estimated, preliminary estimates of the maximum flood from any catchment may be made by averaging the "weighted" daily rainfalls of the stations in and around the catchment for a somewhat longer period, and from the curve thus obtained selecting the greatest average fall for any included period equal to the period of total contribution. The run-off appropriate to this fall is, at first sight, the maximum. A shorter period, however, may give as large, or a larger, run-off, although the entire area has not contributed; thus, consider an area, A, with three days as the period of total contribution. The precipitation for three successive days was 2.4, 7.9, and 2.2 in., respectively, or an

average precipitation of 4.2 in. (0.35 ft.) per day. For such a rainfall a run-off coefficient of $\frac{5}{7}$ was suitable, under the conditions, giving $\frac{5}{7} \times 0.35 \times A = 0.25\ A$ cu. ft. per day, for the short period of the peak of the flood.

On the other hand, one day's rain in the part of the area nearest the outlet, where the stream velocities are greatest, will be gathered in many cases, from more than one-third (say, five-twelfths) the area. For the intense rainfall of 7.9 in. (0.658 ft.) per day, the coefficient of run-off is taken as $\frac{9}{10}$, and the run-off is, $\frac{9}{10} \times 0.658 \times \frac{5}{12} A = 0.247$ cu. ft. per day, at the peak. It is obvious that, since a period of one day gives nearly the same result as a period of three days, in this preliminary estimation, a complete diagram for the whole period is necessary.

If there are small outlying portions on the far side of the catchment, the actual result would be that the calculation for one day would give the greater flood. These special tests are often worth while. In the present connection their main significance lies in their obvious superiority to any formula and their equally obvious inferiority to the hour-by-hour graphical computation.

In some cases of direct calculation from the period of total contribution, the maximum flood is that found by calculating for an area which is the catchment less any extreme outlying portions. In the case of a long and narrow catchment it may be necessary to treat the whole of it, step by step, in the same way, because the maximum flood may be that due to a short period of heavy rainfall on quite a small proportion of the catchment, next to the outlet. The writer has found that this is especially possible in North-eastern Brazil, where the heaviest rainfalls in any day are often those due to intense falls of much less than a day compared with the average for one day.

It is clear that a formula is of small value compared with calculations based on the period of total contribution, and it is even more certain that, for final estimates, no other system of computation is in the same class with direct calculations carried out as described herein.

The accuracy of the computations depends upon the different periods of "delay" being adequately represented. Therefore, there must be division into areas, the centroids of which are at various distances from the outlet. Usually, a good method is to define, approximately, nearly whole catchments for the more distant parts of the area and part catchments for the sections nearer the outlet. The latter should be separated laterally by known or assumed water partings and, on the other two sides, by lines corresponding to those dividing the zones. In zoning, the lines are irregular, so as to give approximately the same "delay" for the entire zone. Division of the catchment into radial sectors fails to utilize the most important feature of division into elements, and is only a little more accurate than a crude whole-area computation, yet such a crude computation is more accurate than any formula, since it allows of the use of smooth curves of rainfall intensity and changing values of the run-off ratio.

The authors have made a very useful contribution to the study of the subject, but, in the writer's opinion, they have failed either to disestablish the rational method or to establish new formulas. First, however, it should be noted that their opinion that the use of empirical formulas is not now to be recommended, is unacceptable—as so expressed without qualification—to those who, like the writer, have found them, in their simplest forms, to furnish very useful comparative data, not otherwise available.

As regards the description of the detail method, this is fundamentally that of the writer, who, for preliminary calculations, or in the absence of rainfall data sufficient to draw the isoplanes for successive days, or shorter periods, uses a rainfall intensity curve. That shown in Fig. 1 is a good characteristic curve very similar to those drawn from data of storms in England, India, and Brazil. It is not, however, a fair presentment of the (detail) rational method, which the writer is defending, because rain-gauge data are used (or should be used) whenever available. Again, in Table 1, the bands, a_1, a_2 ... a_n, correspond to zones, but irregular lines based on estimates of rates of flow must give more accurate results than parallel straight lines. Such irregular lines, therefore, are drawn (or should be drawn), when the area does not lend itself to division into elements which approximate to separate catchments. The "Statement" preceding Table 3 certainly does not apply to the writer's method, by which, as nearly as possible, the rainfall of every small element and the run-off corresponding to it are given hour by hour.

There is, however, a broader and more significant consideration, which the authors have overlooked, namely, that the method really gives the nearest estimation of maximum run-off, whether it is due to the rainfall of the whole period of contribution over the entire catchment, to that of a somewhat shorter period (salients being excluded), or to that of a fraction of the period, such as one-third.

A reversion to the use of formulas, as advocated by the authors, would give, therefore, less reliable estimations, because no formula relates, as does the diagram—hour by hour in time and element by element in area—the factors of "pre-soakage," run-off in proportion to rainfall, rates of flow, and the least inaccurate representation possible (by a smooth curve) of the rainfalls of days and those of shorter periods for which it may have been measured. In any case, engineers are not likely to use to any considerable extent a method involving calculations the character of which is indicated by Appendix I.

G. S. TAPLEY,[22] ASSOC. M. AM. SOC. C. E. (by letter).[22a]—The authors have made perhaps the most thorough analysis of the problem of run-off which has appeared to date. They have pointed out a slight fallacy in the present detail method. They have defined several factors (notably slope) which have not heretofore been clearly understood, and, with good logic, have discussed other factors. Only superficial criticism can be offered.

[22] With City Engr.'s Office, Los Angeles, Calif.

[22a] Received by the Secretary, August 11, 1931.

Probably the least easily understood feature of the analysis relates to P, the factor which introduces into the equation the effect of shape of area and manner of concentration. Apparently, P is a dimensionless factor obtained by trial solutions of sample cases. The writer would like to see a demonstration proving that two areas for which P is equal are geometrically similar as to shape of area and arrangement of the system, because this is implied.

A little confusion in notation exists where the factor, P, is introduced forming Equation (19). Possibly the authors had in mind the desirability of keeping the number of symbols at a minimum. In Equation (14), $c\sqrt{r}$ represents average conditions, whereas the symbol, Q, in the same equation represents the final value. In Equation (18) if Q still represents the final value then $c\sqrt{r}$ must also represent conditions at the final stage, since $c\sqrt{r}$ and Q are coincident factors at a given point. In Equation (19), $c\sqrt{r}$ represents average values, although Q is the final value. In this equation, P has been introduced to convert the final value of $c\sqrt{r}$ (corresponding to the final Q) to the average value.

Although the idea involved in introducing P is simple, it may not at once be obvious. Adopt the notation c', r', Q', v', and s' for elements at any point in the system; c, r, Q, and V for elements at the lower end; and c_a and r_a for average values. Then, at any point,

$$(c'\sqrt{r'})^{0.5} = N\ (Q')^{\frac{1}{2}} \qquad (78)$$

Also, $c'\sqrt{r'} \propto v'$; hence, $\dfrac{c'\sqrt{r'}}{c\sqrt{r}} = \dfrac{v'}{V}$, and,

$$c'\sqrt{r'} = \frac{v'}{V}\, c\sqrt{r} \qquad (79)$$

Therefore, at the average point,

$$c_a\sqrt{r_a} = \frac{v}{V}\, c\sqrt{r} = P\ c\sqrt{r} \qquad (80)$$

but, at the final point, it follows, from Equation (78) that,

$$(c_a\sqrt{r_a})^s = P^s\, N^{2s}\, Q^{0.25s} \qquad (81)$$

which corresponds to the authors' Equation (19).

In defining average slope as a function of time the authors have cleared up a point which has been puzzling storm drain designers. It has been the custom to consider the slope of an area as being determined by certain geometrical dimensions of it, but, since the authors have pointed it out, it is realized that an essential element of slope is time.

It is believed that the derivation of Equation (36) would be more lucid if a mathematical notation were used. As the authors have stated, $t \propto \dfrac{1}{F^2 S^{\frac{1}{2}} Q^{\frac{1}{2}}}$, in which the factors represent conditions at any point, S being the independent variable. That is,

$$t = u\, \frac{1}{F^2 S^{\frac{1}{2}} Q^{\frac{1}{2}}} \qquad (82)$$

Consider any particular interval, Δt_n, of time:

$$\Delta t_n = u \frac{L_n}{F^2_n S^{\frac{1}{2}}_n Q^{\frac{1}{2}}_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

The total time is,

$$T = u \sum \frac{L_n}{F^2_n S^{\frac{1}{2}}_n Q^{\frac{1}{2}}_n} = u \frac{1}{S^{\frac{1}{2}}_a} \sum \frac{L_n}{F^2_n Q^{\frac{1}{2}}_n} \quad \ldots\ldots\ldots (84)$$

in which, S_a is the average slope. Hence,

$$\frac{1}{S^{\frac{1}{2}}_a} = \frac{\sum \frac{L_n}{F^2_n S^{\frac{1}{2}}_n Q^{\frac{1}{2}}_n}}{\sum \frac{L_n}{F^2_n Q^{\frac{1}{2}}_n}}$$

from which Equation (36) follows.

In deriving Equation (39) it is believed that a different definition of m is required in order to permit a logical following through of the development. If, for example, m is defined as the percentage of $(i - z)$ stored, a consecutive development results. Since $(i - z)$ equals the number of cubic feet per second per acre (storage plus run-off), $m\ (i - z)$ equals cubic feet per second per acre, storage. Hence, $(i - z) - m\ (i - z)$ equals cubic feet per second per acre, run-off; or, $A\ [(i - z) - m\ (i - z)]$ equals cubic feet per second, run-off. Since, however, CiA equals cubic feet per second, run-off,

$$CiA = A\ [(i - z) - m\ (i - z)]$$

or,

$$C = \frac{(i - z) - m\ (i - z)}{i}$$

from which Equation (39) follows.

In the paragraphs following Fig. 1, and at several other points, the authors state, "* * * it is the evident intention [of users of the detail rational method] to assume a uniform intensity over the entire area throughout the time of concentration * * *." The idea involved, however, is somewhat obscure.

At any point in the system the discharge is computed from $Q = CiA = Ci \Sigma a_1 + a_2 + \ldots a_n$; that is, each element of area is multiplied by the same value of i. Hence, since the various elements of area, a_1 a_2, etc., did not receive their contributions of rainfall at simultaneous times, i must not have varied with time, and, therefore, must have been constant.

The use of uniform intensities entails the assumption that the conduit in the upper part of the system will not be flowing full. Hence, as the authors have now made apparent, the use of the detail method involves fictitious velocities.

The authors' formulas have obvious applicability as improvements over existing empirical formulas for computing flow at a point. For designing a system, however, the proposed method is not a short cut, since, of course, the conduit would have to be designed in detail from inlet point to inlet point.

W. I. HICKS,[23] ESQ. (by letter).[23a]—This paper is an exhaustive, logical, and thorough résumé and analysis of fundamental and mathematical data on the subject. It throws new light on various hitherto obscure and neglected basic principles and tends to close up several mistaken channels of thought which have been established by erroneous conceptions of the meaning of the available fundamental data.

The authors make an especially noteworthy point, that the quantities computed under the detail method may be exceeded under the circumstance of the peak of rainfall intensity becoming synchronized with a large value, CA, of some tributary area. The writer disagrees with the authors in their exclusion of the consideration of this circumstance from the computations.

The study of factors governing the selection of the coefficient of run-off, C, is valuable and will aid in the planning of experiments and analyzing data therefrom. The logic of deriving coefficients of both storage and absorption, as given in Table 7, from the experimental work of Ivan E. Houk, M. Am. Soc. C. E., is not apparent, as the coefficients are distinct from each other and must be evaluated by separate experiments.

Comments on the detail rational method of computation regarding, in particular, the method of arriving at the time of concentration by means of varying conduit or channel velocities, bring out several possibilities, namely, that the time of concentration may be equal to the time derived in the detail method or may be much greater, dependent on the slope.

The formula derived for the run-off is built up in an ingenious manner, and is correct under the assumptions. However, some of the assumptions are not always correct, or they are difficult of application.

The assumption of uniform rainfall for the time of concentration is not acceptable for short times of concentration. Examination of storms in Los Angeles for a number of years shows a distinct tendency for the peak rainfall to occur after the middle of the storm. Approximately 70% of the precipitation for 1 hour occurs within an interval of 10 min. The single peak of precipitation will synchronize automatically with the peak of the CA quantities contributing and will produce a higher peak run-off than a uniform rate of rainfall. A large percentage of storm drains built by municipalities falls within the time range of about 60 min. The detail method is as much at fault as the formula method in this respect.

In the derivation of the formula, the factor, F, is based on the conduit running full. The rainfall is assumed to be uniform in intensity for the time of concentration. At each point of concentration above the point under consideration, the uniform intensity is of higher value, and produces a conduit of correspondingly larger capacity per unit net area. The conduit is constructed with a difference of from 3 to 6 in. between sizes. The designer ordinarily selects the higher dimension as a factor of safety; furthermore, where slopes are flat, he selects a conduit to give a minimum velocity to prevent silt deposit; this often leads to an oversize. Losses for curves and

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[23a] Received by the Secretary September 2, 1931.

structures along the main line must be subtracted from the gross head available in the fall of the surface grades. The result is that for all conduits above the point of concentration, and under the uniform intensity assumption, the friction slope for the full conduit is flatter than the surface grade.

In areas with steep slopes, the pipe will not run full at all points, which makes the value of F incorrect as tabulated. Inspection of the values of F under partly full flow conditions indicates an increased value over that for full flow for closed conduit, and a reverse condition for open-channel sections. In the flatter terrain, where it is possible for back pressure to fill the conduit, the formula is possible of solution with the given tabular values of F. However, to be technically correct, it is necessary, for each point of concentration, that detail tabulation of the friction loss of each length of conduit be made, together with other head losses. An aggravated example of the latter case occurs when an existing storm drain on a flat slope (and subsequently relieved of a part of its normal tributary area by an intercepting drain) is to be examined for capacity under the reduced area. The extreme oversize of the upper part of the conduit makes the application of the formula of doubtful value. A compensating factor, which applies, practically, to all but the last-mentioned case, is the low power of S used in the formula. With proper judgment in selecting S, more correct results are obtainable.

The factor, $P\left(=\frac{v}{V}\right)$, has been tabulated by the authors by a detailed analysis of numerous types of water-sheds. While it is thus arbitrary, a number of tests of the values reveals comparatively small differences from results obtained by detailed analysis, when the value of C is the same for each computation.

The analysis of the factor, C, points to the probability of variation with time and intensity. To handle this variation in the formula, the value of CA must be estimated from a detailed study of the area. On areas containing a large percentage of pervious soil, the study will be difficult, due to the fact that the coefficient of retention varies with the rate of precipitation for any instant of time and not with the average rate of precipitation for a period of time.

The use of the formula for general municipal storm-drain design does not seem advisable. To be able to use it intelligently, the designer must thoroughly understand its derivation and ramifications, and generally must make an analysis equal to that in the detail method. The careful judgment necessary to achieve correct results is likely to be slighted in the desire for a quick answer. The authors do not recommend it for such work. For estimating work, it can be used for shortening time of computations. For bridges and short culverts, the factors of loss other than friction are relatively so much greater, that the formula can be applied for final design.

By using Method No. 2 (which consists of a trial computation assuming a conduit laid up the main channel to the farthermost ridge point), a correction for time determined by an examination of the nucleus (or initial area

which has no conduits), and a recomputation with the corrected time in the equation, $Q=CiA$, a quick check can be made at controlling points of the tabling by the detail method.

The purpose of the authors, to construct a formula capable of arriving at a close approximation of the run-off from a large area, has been attained in that the formula more or less duplicates the results obtained by the detailed rational method as now practiced. As set forth in the foregoing remarks, it has certain difficulties of application which require close analysis and more careful judgment in handling than the detail method.

The writer is inclined to give the greater weight to their recommendation following Table 6, urging "a more unified effort on the part of the profession not only toward a thorough analysis of present available data, but also future experimentation along lines following a preconceived plan." Drainage engineers have about arrived at the end of their resources in theory and need data to prove or disprove it.

CARL H. REEVES,[24] M. AM. SOC. C. E. (by letter).[24a]—The writer is deeply appreciative of the great amount of work and study shown in this paper. That which stands out most prominently is the logical sequence of the analysis as shown in Part I, and the resultant derivation of the formulas in Part II, bringing to the fore that most important of all factors the determination of C, as given in Equation (39).

Engineers have apparently let an innate tendency to worship precedent lead them in the line of least resistance in determining run-off. In other words, they have taken for granted all that has been said and written and have blindly proceeded to apply it to any and all cases without due regard to the facts in any particular case; and by "the facts," it is intended specifically to mention that intensity of rainfall in its relation to the run-off coefficient has been a sadly neglected quantity in many calculations. It would appear that this is the most important factor in determining the value of C, or the coefficient of run-off.

Equation (39), with its accompanying explanatory sentences, is so patent that it is difficult to believe that this feature has been neglected or ignored so generally through the years. The application of this one idea alone is a distinct contribution to the Engineering Profession, and will simplify and clarify all long drawn-out and complicated tabulations which have been based on a value of C determined only by the presumed best judgment of the designer, or, in many cases, his best guess.

Fundamentally, the rainfall curve as generally used is the "right bower" to the factor, C, as determined in this paper. The analysis of the rainfall curve as applied in the so-called rational method is at once both illuminating and the cause of more rational thinking along these lines. As previously mentioned, engineers have been blindly following what they thought was a

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[24a] Received by the Secretary October 3, 1931.

use of the rainfall curve giving intensities which represented the sequence of an actual storm. It is quite plainly shown that they have not been applying intensities which followed the law of the curve in use.

The writer wishes to congratulate the authors of this paper for the simplification and clarification of the many factors affecting run-off; but, withal, it would be well not to forget the really definite determination of the factor, C, as shown in Equation (39). The great benefit that this paper and its deductions will mean to engineers in private practice, railroads, State highway engineers, flood-control officials, and others having to do with an economical as well as a safe design of culverts, watercourses, storm drains, etc., cannot be over-estimated.

JOHN M. KEMMERER,[25] ASSOC. M. AM. SOC. C. E. (by letter).[25a]—The writer is interested in the authors' discussion of the use of empirical formulas, which they do not recommend. The paper definitely points out the hazards in the use of such formulas and clearly demonstrates that they are little better than a "rule-of-thumb method."

Factor C, based upon experimental data, has always required, and will continue to require, the very best judgment of engineers. Under the heading, "Analysis of Experimental Data," combined with the conclusions, suggestions, and comments of Part I, the authors present a thorough analysis of the factor. Equation (39) should prove useful as time goes on, and as data are collected for analysis.

The fact that the run-off coefficient increases with intensity is well illustrated in the paper by Equations (5), (6), (7), (8), and (9). Conclusions (13 and (14), in the writer's opinion, are logically drawn and should receive the attention of engineers, especially those engaged in experimental work.

The analysis leading to Equation (24), a master equation, is clear, and new factors are included, which make it flexible and generally applicable to all areas. The use of this equation and the new factors requires a greater familiarity with the subject of run-off than has been customary with engineers, but this should lead to better professional results.

Messrs. Gregory and Arnold are to be commended and congratulated for the work which they have done in connection with the preparation of their excellent paper. It is manifest that much time and careful thought have been given to the problem. The writer has often felt the need of some ready adequate method of determining run-off and, therefore, commends the authors, particularly for their painstaking work which, in his opinion, has resulted in two distinct contributions: (*a*) The most complete analysis of Factor C yet presented; and (*b*) an equation (Equation (24)) which includes variables not before recognized, is universally applicable, and is rational in derivation.

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[25] Civ. and Hydr. Engr., Whittier, Calif.

[25a] Received by the Secretary October 5, 1931.

LEONARD L. LONGACRE,[26] ASSOC. M. AM. SOC. C. E. (by letter).[26a]—In the consideration of a new formula the first concern is whether or not the derivation is based on a logical foundation. The formula presented in this paper is derived by starting with the fundamental equation, $Q = CiA$, as used in the detail rational method. The derivation follows in logical sequence until the factors, F and P, are introduced.

There can be no question concerning the velocity in the main flow channel affecting the time of concentration and, consequently, the discharge. The velocity is affected by the shape, and type of the flow channel, and the factor, F, is introduced to allow for this variation. The authors have presented a novel method of determining the value of F.

The writer is unable to explain why $\frac{(c\sqrt{r})^{0.5}}{Q^{\frac{1}{3}}}$ should have a constant value in Equation (18). Factor P makes a sudden and unannounced appearance in Equation (19), and it is unfortunate that the reader must stop and search in separate portions of the paper before an idea may be gained of the authors' intention in introducing this factor.

The arrangement of the collecting channels, which are influenced by the shape of the area, determines the quantity of water flowing at any certain point in the main flow channel. The quantity flowing in the main channel affects the velocity, and, as stated, this affects the time of concentration and discharge. It is evident that Factor P is introduced to allow for this variation in the arrangement of the collecting channels.

The writer is somewhat confused by the discussion just preceding Equation (19). It is evident that the expression, $c\sqrt{r}$, is used first as an abstract quantity to determine the value of N, and then is introduced in Equation (19) as a measure of the average velocity of flow through the main channel. With a constant slope the velocity varies directly as the term, $c\sqrt{r}$, which can be expressed as a function of the factors, P and N. In Equation (19) Factor P could then be defined as the ratio of the average quantity flowing in the main channel to the quantity at the outlet. The authors have defined the factor, P, as the ratio of the average velocity in the main channel to the velocity at the outlet. As the velocity varies with the quantity, it can be used as a measure of Factor P as well as the quantity. The method of arriving at definite values of Factor P outlined in the paper is somewhat involved with assumptions. Before a general acceptance of the values of the factor, P, listed in Tables 3 and 4, the Engineering Profession will have to confirm these values in actual use.

It is the writer's opinion that all the factors included in Equation (24) have a direct effect on the run-off, and should be considered in any run-off problem.

The authors have given an adequate discussion on the shortcomings of previous empirical formulas, and have stated forcefully the advantages of their own. The writer does not believe the refinement of computation, as

[26] Alhambra, Calif.

[26a] Received by the Secretary October 6, 1931.

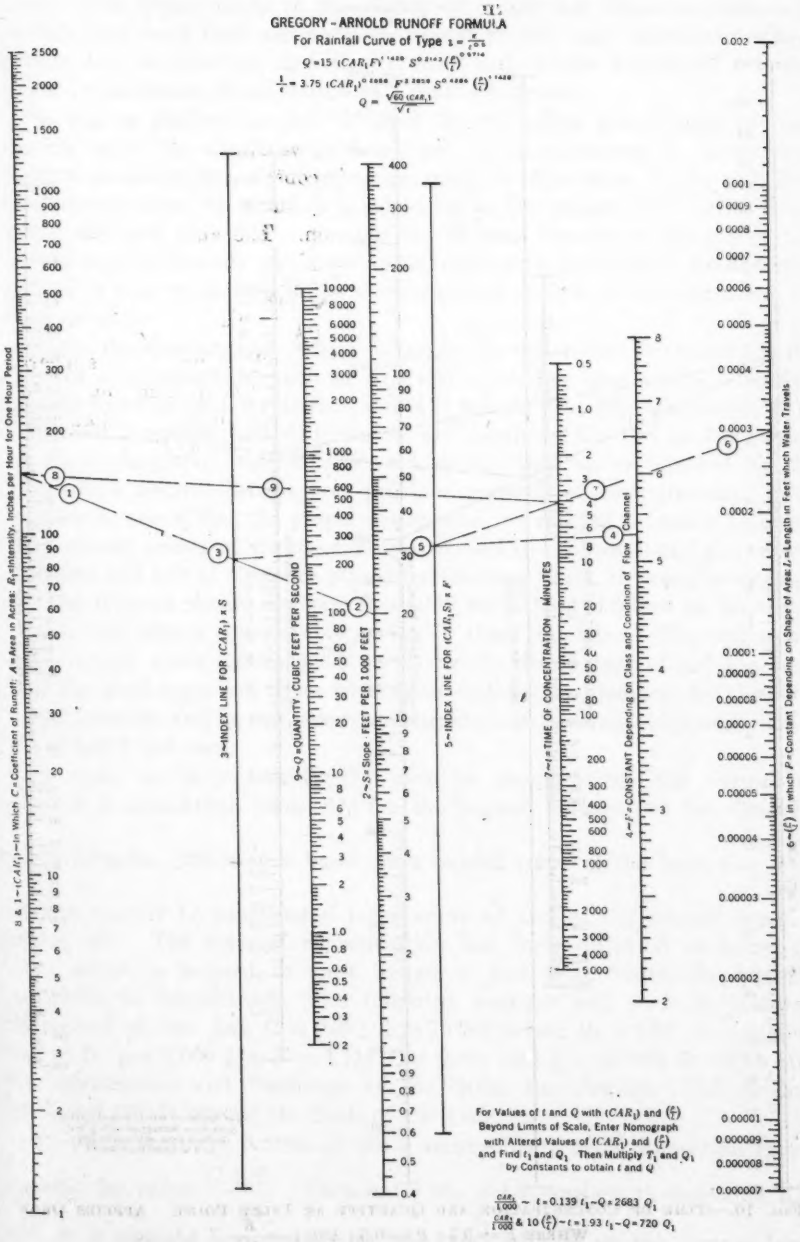
GREGORY - ARNOLD RUNOFF FORMULA
For Rainfall Curve of Type $i = \frac{K}{t^{0.5}}$
$Q = 15\ (CAR_1F)^{1.1429}\ S^{0.2143} \left(\frac{F}{t}\right)^{0.5714}$
$\frac{1}{t} = 3.75\ (CAR_1)^{0.2858}\ F^{2.2858}\ S^{0.4286}\ \left(\frac{F}{t}\right)^{1.1429}$
$Q = \frac{\sqrt{60}\ (CAR_1)}{\sqrt{t}}$
8 & 1 - (CAR_1) - In Which C = Coefficient of Runoff; A = Area in Acres; R_1 = Intensity, Inches per Hour for One Hour Period
3 - INDEX LINE FOR $(CAR_1) \times S$
9 - Q = QUANTITY - CUBIC FEET PER SECOND
2 - S = Slope - FEET PER 1000 FEET
5 - INDEX LINE FOR $(CAR_1S) \times F$
7 - t = TIME OF CONCENTRATION - MINUTES
4 - F = CONSTANT Depending on Class and Condition of Flow Channel
6 - $\left(\frac{F}{t}\right)$ in which F = Constant Depending on Shape of Area; L = Length in Feet which Water Travels
For Values of t and Q with (CAR_1) and $\left(\frac{F}{t}\right)$ Beyond Limits of Scale, Enter Nomograph with Altered Values of (CAR_1) and $\left(\frac{F}{t}\right)$ and Find t_1 and Q_1. Then Multiply T_1 and Q_1 by Constants to obtain t and Q
$\frac{CAR_1}{1000} \sim t = 0.139\ t_1 \sim Q = 2683\ Q_1$
$\frac{CAR_1}{1000}$ & $10\left(\frac{F}{t}\right) \sim t = 1.93\ t_1 \sim Q = 720\ Q_1$

FIG. 9.—TIME OF CONCENTRATION AND RATE OF RUN-OFF.

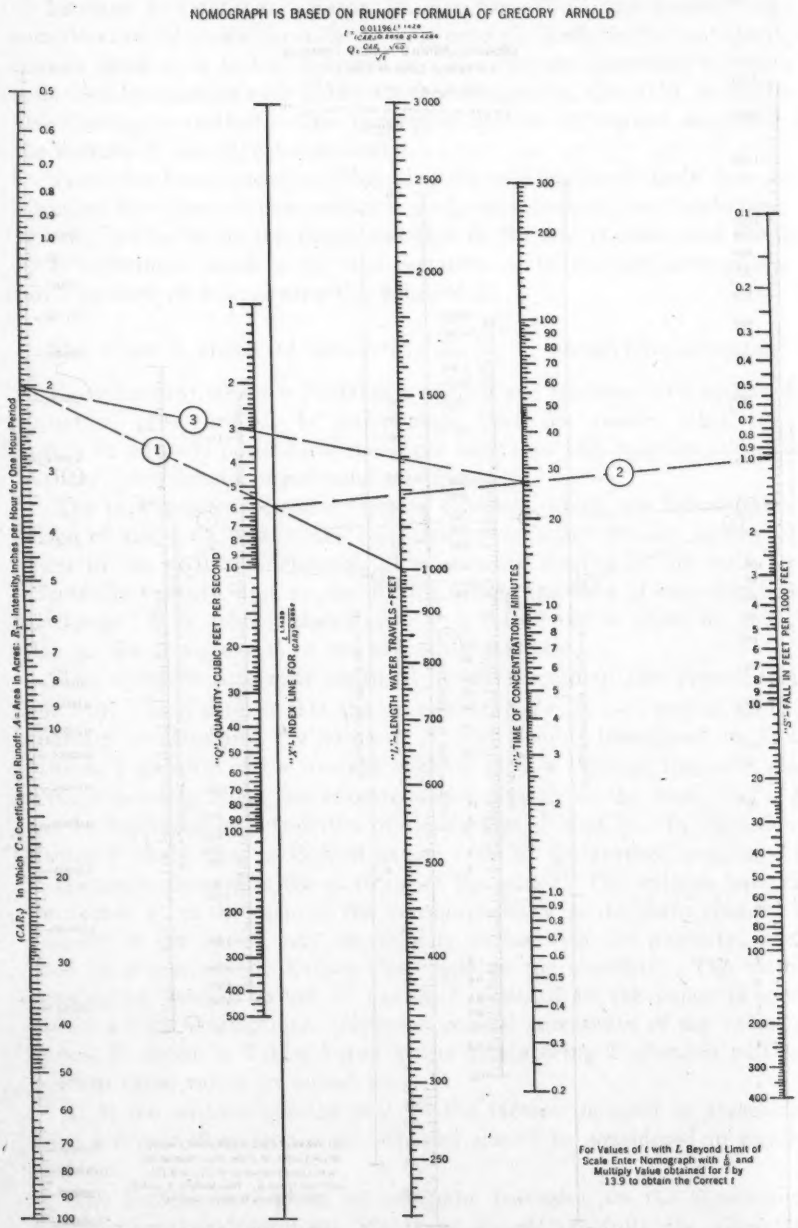
NOMOGRAPH IS BASED ON RUNOFF FORMULA OF GREGORY ARNOLD
(CAR)ⁿ In Which C = Coefficient of Runoff; A = Area in Acres; R = Intensity, Inches per Hour for One Hour Period
"Q"-QUANTITY - CUBIC FEET PER SECOND
"X"-INDEX LINE FOR
"L"-LENGTH WATER TRAVELS - FEET
"t"-TIME OF CONCENTRATION - MINUTES
"S"-FALL IN FEET PER 1000 FEET
1
2
3
For Values of t with L Beyond Limit of Scale Enter Nomograph with L/13.9 and Multiply Value obtained for t by 13.9 to obtain the Correct t

FIG. 10.—TIME OF CONCENTRATION AND QUANTITY AT INLET POINT. APPLIES ONLY WHERE $F = 5.5$; $P = 0.5$; AND $i = \frac{K}{t^{0.5}}$

suggested under "Method No. 1," is justified where seven variables are to be chosen. The errors made in the choice of values for these variables will certainly be greater than any accuracy gained by this more elaborate method. Neither does he consider that the formula will replace the detail rational method in the actual detail design of a drainage system.

Its use, as outlined under "Method No. 2," offers possibilities in conjunction with the detail method. Where it is necessary to investigate different combinations of collecting channels to determine which will give the maximum flow, or where it is advisable to investigate whether or not a part of the area may give a greater run-off than the whole, the use of the Gregory-Arnold formula will save considerable time and labor. Another use to which it may be applied is the determination of time of concentration for initial areas.

Where the discharge at certain points in the water-shed is needed for the design of some structure, such as a culvert or bridge, on a stream where no adequate hydrograph is available (and it is seldom that any satisfactory data on rainfall intensity and distribution are available for use in connection with the hydrograph), the formula used, as described under "Method No. 2," will prove a decided benefit over previous methods of computations. It is necessary however, that the proper assumption for rainfall intensity be made. If the correct values of various factors are used in each case, and the run-off coefficient and initial time are placed on the same basis, the results obtained from the formula should compare favorably with those obtained by the detail method, and with a considerable saving in time and labor. The writer has tabled several areas (about 3 000 acres) by the detail method and has compared the discharges and times with those obtained by applying the Gregory-Arnold formula, and he was somewhat surprised at the close agreement, which was within 3 per cent.

In order to save considerable time in computations the writer has prepared a nomograph (Fig. 9), for the general solution of the Gregory-Arnold formula. While it is based on a rainfall curve of the type, $i = \frac{K}{\sqrt{t}}$, one can readily be constructed for a curve of any of the general types in present use. The time of concentration has been evaluated as being the factor which is desired in most instances, and from which the quantity can easily be determined. The following example will serve to illustrate the method of use: Let, $C = 0.30$; $A = 10\,000$ acres; $R_1 = 1.23$ in. per hour; $S = 10$ ft. per 1 000 ft.; $F = 4.71$; $P = 0.55$; and $L = 35\,000$ ft. The time of concentration and discharge at the outlet are desired. The product, $CAR_1 = 3\,690$, is beyond the limit of the scale.

In the note at the bottom of the nomograph are given correction factors to apply for using $\frac{CAR_1}{1\,000}$. Then enter the CAR_1-scale with the value, 3.69, and, on a straight line connecting this value with 10 on the S-scale, locate the intersection with the index line numbered 3. From this point, on a line

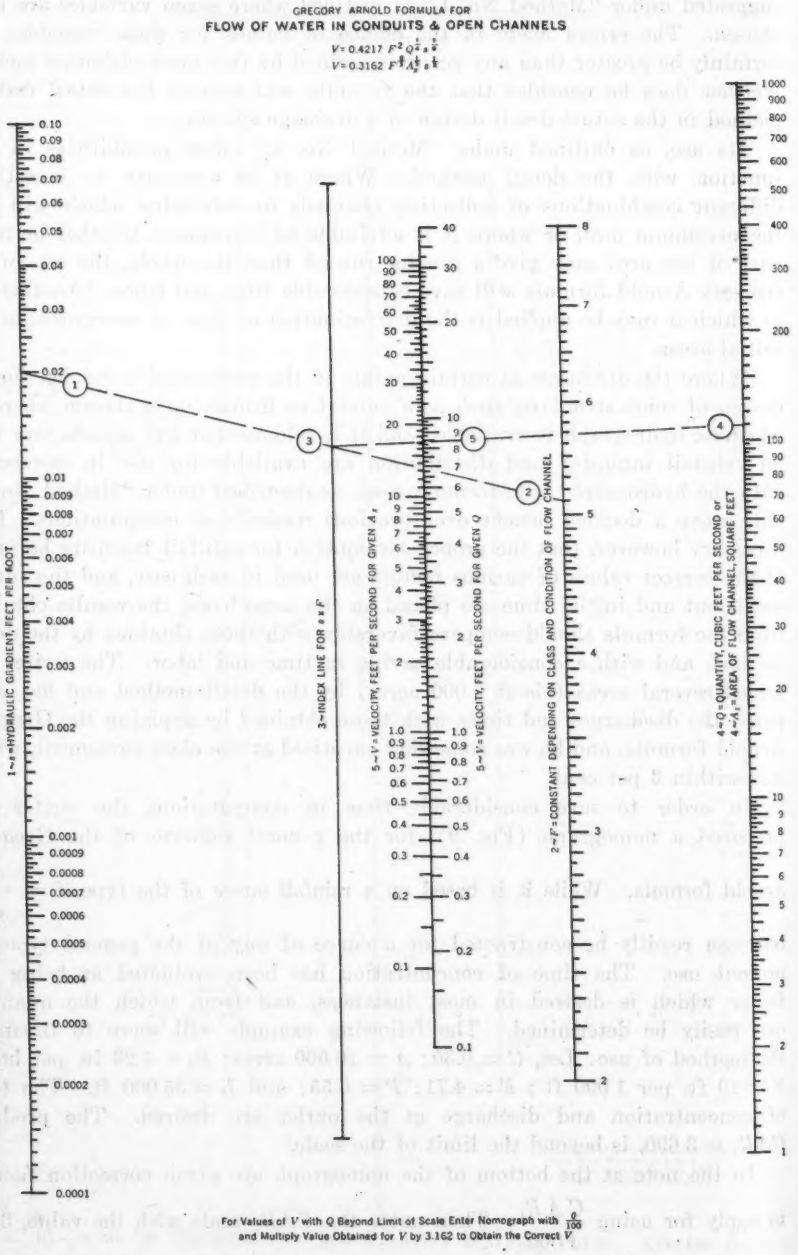
GREGORY ARNOLD FORMULA FOR
FLOW OF WATER IN CONDUITS & OPEN CHANNELS
$V = 0.4217\ F^{\frac{2}{3}} Q^{\frac{1}{4}} s^{\frac{1}{2}}$
$V = 0.3162\ F^{\frac{2}{3}} A_1^{\frac{1}{2}} s^{\frac{1}{2}}$
1 = s = HYDRAULIC GRADIENT, FEET PER FOOT
3 = INDEX LINE FOR s × F
5 = V = VELOCITY, FEET PER SECOND FOR GIVEN A_1
5 = V = VELOCITY, FEET PER SECOND FOR GIVEN Q
2 = F = CONSTANT DEPENDING ON CLASS AND CONDITION OF FLOW CHANNEL
4 = Q = QUANTITY, CUBIC FEET PER SECOND or
4 = A_1 = AREA OF FLOW CHANNEL, SQUARE FEET
For Values of V with Q Beyond Limit of Scale Enter Nomograph with $\frac{Q}{100}$
and Multiply Value Obtained for V by 3.162 to Obtain the Correct V

FIG. 11.

connecting 4.71 on the F-scale, locate the intersection with the index line numbered 5. From this point, on a line connecting 0.0000157 on the $\frac{P}{L}$- scale, read the value of 610 at the intersection with the t-scale. This value will be designated as t_1. From this point, at 610 on the t-scale, on a line connecting with 3.69 on the CAR_1-scale, read 1.16 at the intersection with the Q-scale. This value will be designated Q_1. From these values of t_1 and Q_1 the desired time of concentration and discharge are obtained by multiplying by the correction factors as given; thus, $t = 0.139 \times 610 = 85$ min.; and $Q = 2\,683 \times 1.16 = 3\,110$ cu. ft. per sec.

For some time the writer has been aware of the inadequacy of the usual practice of arbitrarily assuming a period of 5 to 15 min. as the time of concentration for the initial area used in the detail rational method. Actual observations have shown that in many cases these assumed times are grossly in error, particularly in flat country where initial times of 25 min. have been observed by the writer, and no doubt even longer inlet times occur. The Gregory-Arnold formula offers a possibility of determining the inlet time of initial areas for use in the detail rational method. The nomograph, Fig. 10, was prepared to facilitate the use of the formula. The numbers in the small circles give the sequence in which the scales are used. Factor F was given a constant value of 5.5 on the assumption that the water running from the most remote portion of the initial area to the point of concentration, would travel most of the way in a concrete gutter. Factor P was assigned a constant value of 0.50 by assuming that the average initial area is a rectangle, the length being twice the width. Another consideration in using a constant value for P is the fact that the percentage of error in the variation of P will be relatively small compared with the error which may occur in choosing the value of C. Where it is advisable to use different values for the factors, F and P, the nomograph shown in Fig. 9 may be used for finding the time of concentration for the inlet areas.

Equation (50) presents a formula which upon investigation proves to give values agreeing very closely with ones obtained by the use of the Chezy-Kutter formulas when values of $c\sqrt{r}$ are greater than 50. It will prove useful in comparing the hydraulic properties of different types of channels. In Equation (50) replacing Q by $A_x v$, in which A_x is the cross-sectional area, in square feet, of the channel, the resulting formula is:

$$V = 0.3162\, F^{\frac{2}{3}} A_x^{\frac{1}{3}} = S^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (85)$$

In this form, knowing the cross-sectional area of the channel, the velocity can be determined. Equations (50) and (85) can easily be combined on a single nomograph, as given in Fig. 11, and when used in conjunction with a table of F factors, makes possible an easy and rapid solution for the velocity, using either a known area, or quantity, for any channel shape.

In conclusion, the writer wishes to state that he considers the paper worthy of considerable study, and that the reader will be amply repaid for his time by a clearer conception of the fundamentals affecting a rational solution of run-off problems.

G. H. HICKOX,[27] JUN. AM. SOC. C. E. (by letter).[27a]—The paper should be of particular interest and value to those engaged in the design of storm sewers or land drainage systems having regular outlet channels. Its value lies in eliminating the error, inherent in the common method of analysis, of using a velocity and total time of flow in the outlet channel which do not correspond with the quantity flowing. In considering a storm over a large area, the smaller portions are assumed to be contributing their share at the same high velocity as their own maximum flow. Since the small area contributes to the maximum discharge for the entire area at a lesser rate than its own maximum discharge, the velocity of flow is less. This assumption of too high a velocity leads to shortening the time of concentration, increasing the rate of run-off, and, consequently, increasing the size of outlet channels, which increases their cost. The formulas proposed by the authors should do away with this unnecessary cost and should prove of great benefit.

The formulas should be used with caution on areas greater than 500 sq. miles. Economic and other features being equal, the rainfall curve used for a large area should not be the same as for a small one. The curve for areas of a few square miles may be plotted from records made at points; but when an area of 500 sq. miles is considered, as in Example 7, the curve so made no longer applies. A storm lasting for 4.4 hours with an intensity of 0.933 in. per hour at the point of observation would much less frequently cover 500 sq. miles for that length of time with the same intensity. The rainfall curve made for any point may represent equally well all points within the basin, but unless the intensity for the time of concentration occurs simultaneously at all points—which is extremely improbable on an area of this size—the average intensity on the basin for the period is less than that at the point. Precipitation on 500 sq. miles would necessarily be more intense at some points than at others. As the rainfall curve is made up of the maximum records at points, it follows that an estimate of discharge for the area, based on such a curve, is excessive. Table 15, prepared from data given in

TABLE 15.—RATIO OF AVERAGE RAINFALL OVER A GIVEN AREA TO MAXIMUM OBSERVED RAINFALL WITHIN THE AREA, FOR STORMS OF 1-DAY'S TO 5-DAYS' DURATION.

Drainage area, in square miles.	Coefficient.	Drainage area, in square miles.	Coefficient.
0	1.00	4 000	0.76
500	0.90	6 000	0.72
1 000	0.86	8 000	0.70
2 000	0.82	10 000	0.67

Part V of the Technical Reports of the Miami Conservancy District, presents coefficients by which to multiply the maximum precipitation observed within a river basin, to obtain the average over the basin (the values are probably slightly high, as it is assumed that the configuration of the storm coincides with that of the basin).

[27] Rock Island, Ill.

[27a] Received by the Secretary October 15, 1931.

The writer is not convinced that the rational formula is ready to displace the empirical formula for the estimation of floods. Empirical formulas are subject to much abuse, being used far afield from the data on which they are based. It is the indiscriminate use of empirical formulas by engineers not familiar with their derivation and limitations, which has brought them into their present disfavor. An empirical formula for the 50-year flood, carefully based on data obtained in the vicinity where it is proposed to be used, and applied only within the range of these data, or moderately extrapolated, is probably superior to the rational formula. The element of frequency enters in connection with economic considerations, as it is seldom economical to design for the absolute maximum. The engineer is not so much concerned with the maximum flood as with that flood which may be expected once in, say, 50 years.

The rational formula assumes an average rainfall over the time of concentration and a uniform distribution over the basin. In general, floods are not produced by these conditions. They are the result of intense localized storms lasting for a fraction of the time of concentration; light, warm rains on snow-covered, frozen ground; and, occasionally, the conditions postulated by the formula. If these conditions, however, only produced the 50-year flood, it would still be impossible to evaluate it on the basis of the 50-year rainfall because of the uncertainty in the value of the run-off coefficient. On stream basins in the Middle West, the value of C may vary more than 100% for summer rainfalls of the same average intensity, due solely to the difference in saturation of the soil. Evidently, only a fraction of the 50-year rainfalls produce 50-year floods, as the soil is more often than not insufficiently saturated to produce maximum run-off. The problem is further complicated by the occurrence of floods due to intense local storms and the spring floods due to melting snow. The empirical formula, however, being based on recorded floods, automatically considers all these factors and groups them into two or three coefficients and exponents.

The Iowa River Basin above Iowa City, Iowa, may be taken as a fair example for comparison of rational and empirical formulas. Its drainage area is 3 140 sq. miles; channel length, 261 miles; and average slope, S, 0.425. The value of P may be taken as 0.65 and that of F as 4.0. The time of concentration is approximately 100 hours. Estimating the 50-year rainfall intensity from the charts[28] prepared by Merrill M. Bernard, M. Am. Soc. C. E., $i = \frac{75}{t^{0.8}}$, or, for 100 hours, $i = 0.715$. Multiplying this by a factor taken from Table 15, $i = 0.78 \times 0.715 = 0.558$. The value of C for the Iowa River Basin is probably in excess of 0.35. Using these values, the rational formula gives a discharge of 59 700 cu. ft. per sec.

F. A. Nagler, M. Am. Soc. C. E., has proposed[29] an empirical formula,

$$Q = 210\,M^{\frac{3}{4}} \qquad (86)$$

for the magnitude of the 50-year flood to be expected on Iowa streams, in

[28] *Proceedings*, Am. Soc. C. E., October, 1930, p. 1847.
[29] "Survey of Iowa Floods," *Proceedings*, Iowa Eng. Soc., 1928.

which, Q is the discharge, in cubic feet per second, and M is the drainage area, in square miles. By Equation (86), the discharge is 45 000 cu. ft. per sec. Floods greater than 35 000 cu. ft. per sec. have occurred three times at Iowa City since 1850, the largest being 50 000 cu. ft. per sec. The discharge given by the empirical formula is seen to fit at least as well as that by the rational formula (which, in this case, is 33% greater), and to involve much less labor.

The maximum precipitation on the Iowa River Basin is probably about 10 in. in 4 days, or approximately 0.1 in. per hour. With $e = 0.8$ and $C = 0.4$, the maximum flood by the rational formula is 86 900 cu. ft. per sec., a flood which is 93% greater than the 50-year flood and obviously beyond the ordinary considerations of economic design.

DONALD M. BAKER,[30] M. AM. SOC. C. E. (by letter).[30a]—The ready acceptance and use of familiar methods and formulas in design and computations, without an intelligent consideration of the limitations to which such formulas are subject, is a practice too common among engineers. A paper such as this, wherein the influences affecting the different factors embraced in the formulas used in the calculation of storm-water run-off are discussed and analyzed, is a decided service. The design of any storm drain system is a complicated process at best, involving many assumptions and a considerable degree of engineering judgment.

The master formula, Equation (24), is a considerable improvement. It goes much farther than older formulas in the inclusion of variables by the addition of terms representing changes in slope and consequent velocity of the main channel above the point of concentration, shape and roughness of this channel, and length and shape of the water-shed. It also allows greater flexibility in determining the period for the selection of the intensity and the variation in type of rainfall curve used. While it is somewhat complicated, nevertheless, in any organization in which much work of this type is done, its use can be facilitated by the development of curves or nomographs.

Of the three factors expressing the fundamental relationship between run-off and rainfall, stated by the authors in the expression, $Q = C\,i\,A$, the area, A, may be determined with a high degree of accuracy, and the intensity of rainfall, i, expressed in inches per hour (or the rate of contribution of water to the water-shed by precipitation, expressed in cubic feet per second per acre), may be ascertained with approximate accuracy in most localities where storm drain systems of any magnitude are to be constructed. Nevertheless, where the formula is to be used in the determination of storm or flood flows, in mountainous sections, or where precipitation records are lacking, much may be left to judgment or imagination.

However, it is the factor, C, which affords the opportunity for widest variation in the use of judgment. General reference to this factor in textbooks and technical papers seems to be based largely upon arbitrary assumptions or upon some previous reference and not upon results of actual experience. No one can give this subject much attention without being impressed with the

[30] Cons. Engr., Los Angeles, Calif.

[30a] Received by the Secretary, October 20, 1931.

paucity of experimental work carried on to establish values for this factor. Storm drain systems are expensive; a complete drainage system in a residential section, designed for a reasonable frequency of excessive precipitation, may cost \$500 to \$1 000 per acre, or \$320 000 to \$640 000 per sq. mile, or even more. The cost of such system, other things being equal, varies directly with the cost of pipes or box-drains used to carry the water, and this, in turn, varies approximately in a direct ratio with the quantity of estimated flood flow, which varies directly with the factor, C. It can readily be seen, therefore, that a variation of 10% in the value of C (say, where it was assumed as 0.45 instead of 0.40, due to lack of definite knowledge) might result in a variation in the cost of a drainage system of from \$50 to \$100 per acre, or \$\$32 000 to \$64 000 per sq. mile. In the light of this, it is surprising that more communities engaged in large programs of storm drain construction have not embarked upon a program of investigations of values for this factor. Nothing like the amount of research has been given to it as to the determination of such factors as proper values for Kutter's n, or constants in weir formulas. Such experimentation should logically be done by a governmental agency, in order that the results obtained could be made widely available. However, there does not appear to be such an agency in the Federal Government which could logically be considered as interested in this subject.

The mathematical proof which the authors offer in substantiation of their conclusions regarding variation of C with intensity, etc., is of considerable interest, and increases the confidence with which these conclusions may be used. A further feature of interest is the derivation of Equation (39), indicating the manner in which the rate of run-off varies with intensity of precipitation. In this equation, m expresses such influences as roughness of the ground surface of the water-shed, and the retarding effect of vegetal cover. Interception of precipitation by trees and brush, or its storage in a mulch of leaf mould, or in a dense grass cover, may result in the collection of a considerable quantity of precipitation, until saturation occurs and allows run-off to proceed. A few investigations along these lines have been made by experimenters interested in irrigation development and in the protection of water-sheds against fires. The quantity, z, used in Equation (39) is influenced principally by the soil type and the slope of the ground surface. During a visit, the writer was impressed by the lack of evidences of surface erosion on the floor of the Valley of Mexico as well as in Florida. Storms of high intensity and rather prolonged duration occur in these localities, but the soil appears to have such a high absorptive capacity that run-off is not evident. Some sandy soils, however, do not have this high capacity, due probably to lack of uniformity of size of grains.

The slope of the ground surface affects the rate at which the surface water concentrates in the small drainage channels and travels in them, but it also affects absorption or percolation. Equation (39) shows that as long as the rate of absorption is greater than the rate of precipitation, or when $z>i$, the factor, C, is a negative quantity, since $m<1$ and no run-off can occur. When $i>z$, however, a surplus collects, which produces run-off. With flat

slopes, velocity of run-off is decreased, depth of run-off is increased, and the resultant head of water on the soil surface also is increased. Using the same nomenclature as the authors,

$$Q = k\,(i - z') \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (87)$$

in which, k = a constant, and z' = maximum unit rate of absorption by soil without superimposed head, constant for any given soil and surface condition; but Q varies as vd.

Let v = velocity; d = depth of water flowing over soil; and k' = a constant; then,

$$Q = k'\ vd = k'\ dc\ r^{\frac{1}{2}}\ s^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

Since $r = d$, approximately, Equation (88) reduces to,

$$Q = k'\ d^{\frac{3}{2}}\ c\ s^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (89)$$

Therefore, $k\,(i - z') = k'\ d^{\frac{3}{2}}\ c\ s^{\frac{1}{2}}$, and,

$$d = \left(\frac{k\,(i - z')}{k'\ c}\right)^{\frac{2}{3}} \frac{1}{s^{\frac{1}{3}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (90)$$

Let z'' = rate of percolation from run-off, and k'' = a constant. Then, since the rate of percolation also varies as the superimposed head,

$$z'' = k''\ d = k''\left(\frac{i - z'}{c}\right)^{\frac{2}{3}} \frac{1}{s^{\frac{1}{3}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (91)$$

and, $z = z' + z''$, which shows that induced absorption, as distinguished from the natural absorption rate of the soil, varies directly with the two-thirds power of the difference between the rate of precipitation and the natural absorption rate of the soil, and also increases directly in a certain manner with the surface roughness, but increases inversely with the cube root of the slope.

In explanation of the fact that a zero value of s would make z'' approach infinity, it may be stated that this relationship only holds when the velocity of flow of the run-off follows the Chezy-Kutter formula.

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DISCUSSIONS

MANUFACTURING CONCRETE OF UNIFORM QUALITY

Discussion

BY MESSRS. STANTON WALKER, AND EDWARD E. BAUER

STANTON WALKER,[13] ASSOC. M. AM. SOC. C. E. (by letter).[13a]—The factors to which emphasis should be given in the design and control of concrete on the job, have been clearly and concisely described in this paper. There are certain points, however, which, it is believed, will be clarified by further comment.

In the paragraph headed "Specifications," it is stated that "sand was required sufficient to yield 15% or more of mortar than was needed to fill the voids in the gravel * * *." In Appendix II, under "Batch Mixing," a considerably different rule for determining the quantity of sand is inferred, as follows: "The specified measure of sand (equal to voids in gravel plus 15% of voids) will ordinarily give only a sufficient amount when a workable mass is well mixed * * *."

The design of proportions by the first rule would result in the use of very much less sand than is ordinarily considered necessary for workability. Taking the highest percentage of "dry-and-rodded" voids reported in Table 3 (37.7%) and assuming a percentage of voids of 35 for the sand and a net water-ratio of 0.9, a mix of about 1:1.3:5.1 is obtained for concrete containing 5 sacks of cement per cu. yd. by the application of the first rule. The application of the second rule results in proportions more nearly those that are ordinarily used although, in the writer's opinion, such mixes are still considerably "under-sanded" for the usual conditions. Using the same assumptions, a 1:1.8:4.4 mix is obtained. The writer would be interested in knowing which of the two rules was actually applied in the design of the concrete. In his opinion neither one of them represents a satisfactory criterion on account of the resulting "under-sandedness" of the mix. A

NOTE.—The paper by William M. Hall, M. Am. Soc. C. E., was published in May, 1931, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1931, by Messrs. Henry B. Seaman, Theodore Belzner, John Sanford Peck, and J. W. Kelly.

[13] Director, Eng. and Research Div., National Sand and Gravel Assoc., Washington, D. C.

[13a] Received by the Secretary September 24, 1931.

volume of mortar equivalent to, at least, 1.65 times the volume of voids in the coarse aggregate will be required to produce satisfactory workability for the usual conditions, and this factor will vary with the size and grading of the aggregate.

A procedure for determining whether or not the test cylinders have plane ends is described in Appendix I. This is one of the most important factors in securing uniform test results and one which is often overlooked. As much of the apparent lack of uniformity in tests of field specimens is due to poor testing technique as it is to actual variations in the quality of the concrete. The writer does not agree that it is sufficiently accurate to determine by the "feel," when a cylinder is rocked on a plane surface, if the end is in satisfactory condition; it may be concave and, if that is the case, it may "feel" perfectly solid. The ends should be tested with a straight-edge or by some equally satisfactory method.

The importance of having the ends of the test cylinders "perfectly parallel" is also stressed in Appendix I. This condition is practically impossible of attainment and reasonable deviation from it is not important if the cylinders are tested in a machine equipped with a suitable spherical bearing block.

Comprehensive information has been given by H. F. Gonnerman, M. Am. Soc. C. E., on these questions.[14] While Mr. Gonnerman showed convex ends to be considerably more detrimental than concave ones, the reductions in strength for the latter condition were important. Cylinders with the top surface inclined showed no reductions in strength for an inclination of $\frac{1}{4}$ in. in 6 in., and only about 5% reduction for $\frac{1}{2}$ in. in 6 in.

EDWARD E. BAUER,[15] ASSOC. M. AM. SOC. C. E. (by letter).[15a]—It is gratifying to note the care that was exercised in securing concrete of high quality on these particular projects. Too often the feeling exists that care and exactness have no place on the construction job; that it is only in the laboratory that these items need consideration. Mr. Hall and his assistants are to be commended for the excellence of their program for supervising the production of concrete.

The writer wishes to emphasize the importance of attending to details in the making and testing of cylinders, in order to secure results that are uniform to any degree. Field cylinders are valuable, but unless certain rules are followed in their making and handling, the results may be erroneous and of no value. The cylinders must also be tested properly.

Mr. Hall has noted that many of his cylinders had to be capped just prior to testing, in order to secure an even bearing surface on both ends. The writer has noted in his testing work that whenever it is necessary to cap a cylinder with plaster of Paris, the crushing strength value is almost always lower than that of cylinders properly made. The ends should be made

[14] "Effect of End Condition of Cylinder on Compressive Strength of Concrete," *Bulletin 14*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill.; reprinted from *Proceedings*, Am. Soc. for Testing Materials, April, 1925, Vol. 24, Part II, 1924.

[15] Asst. Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

[15a] Received by the Secretary October 5, 1931.

plane surfaces when the cylinders are cast in the field. It is difficult not to rock the plate or the cylinder in applying the plaster-of-Paris cap and thus produce a curved surface. The proper making of the cylinders should be given more attention by field men, in general. Cylinders that are patched up in the laboratory can not always be made to give correct results. Cylinders without plane ends should never be tested, yet one occasionally hears of this being done in some laboratory.

In Appendix I the statement is made that the ends should be "perfectly parallel." The writer sees no need for this, since the properly equipped testing machine has a spherical bearing block to be used on top of the specimen to allow for any lack of parallelism in the ends. It would seem that any attempt to secure perfectly parallel ends with an ordinary carpenter's square would be a hopeless task. While no statement is made as to whether or not a spherical bearing block was used, the precautions recommended lead one to believe that it was not. Without a spherical block there can be no positive assurance that the load is distributed over the entire cylinder. Concentration of pressure on a small area will cause the cylinder to fail at a lower load.

According to Appendix I the cylinders were kept in damp sand, the temperature of which may have varied between 60° and 90° Fahr. Tests at the University of Illinois[16] indicate that if the curing temperature after the first day is varied a uniform amount for the entire curing period, the strength is affected about 30 to 35 lb. per sq. in. per degree of change in curing temperature. The concrete in these tests showed a strength of 5 400 lb. per sq. in. at 28 days under standard curing conditions. The 1929 cylinders are fairly close to this figure, so that it may be safely assumed that the range of curing temperatures allowed on the Ohio River projects permitted a maximum variation in strength of 900 to 1 000 lb. per sq. in.

Cylinder strengths as reported vary over too wide a range. If the concrete placed in the structures was of uniform quality, then most certainly some of the cylinders did not receive the proper attention.

[16] See "High Early Strength Concrete," by the writer, *Proceedings*, Am. Concrete Inst., 1929.

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DISCUSSIONS

DESIGN OF LARGE PIPE LINES

Discussion

BY MESSRS. L. J. MENSCH, AND W. P. ROOP

L. J. MENSCH,[14] M. AM. SOC. C. E. (by letter).[14a]—This interesting paper is too abbreviated for American practice. It is based on a number of German articles on the strength of membranes, the authors of which claim to have obtained simple working formulas by making simplifying assumptions in the treatment of the subject as found in Professor Love's "Treatise of the Mathematical Theory of Elasticity." The author accepts their differential equations on trust and then calls his product an exact theory. This latter expression is constantly found in all prominent works on elasticity from Professor St. Venant on, but the writer considers it very misleading and unjustified by tests and experience. Geometry is considered an exact science for the purpose of the engineer (but not for that of the astronomer as Riemann and Einstein have showed), but the new sciences of mechanics and strength of materials are not advanced far enough to treat of exact theories in these branches. Engineers have no exact column theory, nor an exact beam theory, but simply a belief that their hypotheses lead to formulas with a more or less approximation of the truth.

Equations (2) and (3) were obtained by omitting a great many factors from what Professor Love calls the equations of equilibrium of an infinitesimal part of the surface of a membrane. It seems that for the case of a full-circle pipe, simply supported around its periphery, these equations lead, as the author shows, to useful results which agree with the common theory; it should be noted, however, that his assumption of stresses for a continuous pipe on many supports cannot be derived from Equations (2) and (3), but are based on the common theory of bending only. Dischinger used these equations for the design of elliptically shaped roofs, but in such cases the writer found that his stresses deviate as much as 50% from those obtained by the common theory, and although the firm with which he is connected, made large-sized tests to prove to the public the safety of its concepts, the

NOTE.—The paper by Herman Schorer, Assoc. M. Am. Soc. C. E., was published in September, 1931, *Proceedings*. This discussion is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

[14] Gen. Contr., Chicago, Ill.

[14a] Received by the Secretary September 25, 1931.

writer finds that the test objects were largely over-designed, although to the ordinary engineer they appear extremely bold and novel. Let it be stated that similar roofs were constructed in the United States ten to twenty years before they were tried in Germany.

The practicing engineer will not have the time nor the opportunity to look up all the authorities cited by the author and this might lead him to disregard, or look with suspicion upon this valuable paper. The writer is pleased to state that more than twenty years ago he obtained the same results as the author by a much simpler method, and can attest the correctness of Equation (28). He knows that Equation (29) was obtained by a difficult chain of reasonings, with many approximations; it may be correct. He can give, however, some data on the condition of restraint which will simplify the understanding and work of the designer. The rim stresses of $1.82\,\frac{pr}{t}$ become zero at the distance, $x = 0.588\sqrt{rt}$. A reversed maximum moment occurs at nearly twice this distance from the rim, namely, $1.15\sqrt{rt}$, and these reversed stresses amount to $0.4\,\frac{pr}{t}$. In the author's example, $r = 60$ in. and $t = \frac{1}{4}$ in.; hence, the two critical distances are 2.75 in. and 4.40 in., respectively. This shows that there are great changes of stress in a very short distance, involving considerable shear, and this part of the pipe should be carefully designed, with low stresses, which practice was recommended to his students by Professor Grashof more than fifty years ago. In the design of the circular ring disk the consideration of the Lamé stresses might be advisable in most cases.

W. P. Roop,[15] M. Am. Soc. C. E. (by letter).[15a]—This interesting and important paper gives a theoretical analysis of a novel type of support for horizontally disposed, thin-walled tubes. In the section headed, "Direct Stresses in Pipe Shell," solutions are offered for the stresses due to weight of tube and contents, carried by the tube itself, considered as a girder, and also as modified by the tensile stresses in the shell (rim stresses) due to hydrostatic pressure. In a second section these rim stresses receive more detailed study, especially as affected by the supports, consisting of very rigid peripheral stiffeners. The stresses in these rings themselves are considered in a third section, and the paper closes with examples showing the practical advantages of this type of construction.

The mathematical procedure is somewhat formal, but the details could doubtless be readily reconstructed by one who wished to follow the analysis through closely. The summary is excellent. The present comment is confined to references to related work not quoted, and to emphasizing some features which may possibly not otherwise come to the notice of the casual reader.

[15] Lieut., C. C., U. S. N.; Asst. to Sup. Constr. for, U. S. Navy, New York Shipbuilding Co., Camden, N. J.

[15a] Received by the Secretary September 29, 1931.

A structure of circular form draws enormous benefit with respect to its rigidity from its symmetry, and the application studied by Mr. Schorer, while extending to various cases mentioned under "Example and Applications," by no means exhausts the usefulness of such a symmetrical form. As examples, the writer wishes to cite the cantilevers upon which he reported in 1929[16] and the tubes under external pressure discussed by Messrs. H. E. Saunders and D. F. Windenburg in a paper[17] entitled "Strength of Thin Cylindrical Shells Under External Pressure."

It is necessary, of course, that the circular form be adequately maintained, and the peripheral stiffener has been found to be an effective means of doing this. In the cases thus far mentioned, the interior of the shell is left open and the stiffeners are rings, inside or outside. In other cases, however, it is possible to use diaphragms extending entirely across the section of the cylinder. Such, for example, are the transverse bulkheads in a submarine. Although surface ships depart widely from circular sections, the form is approximately cylindrical on a section partly symmetrical. The typical structure is reproduced naturally in the stem of the bamboo plant.

Turning again to Mr. Schorer's paper, it is noted that his numerical example shows stresses of an acceptable intensity in pipe the diameter of which is 480 thicknesses supported only by peripheral stiffeners at intervals of 6 diameters. Contrast such a structure with one of cast pipe, supported by trusses, such as may be seen in many large industrial plants. The writer believes that there is no doubt that savings in weight of the order of one-half or more of that used in continuous structures of traditional type can be made in many cases by close analysis of the requirements of accurately specified loads, and that a structure of continuous character may be built up of plates shockingly thin if due attention is given to their favorable disposition and to their adequate support even at rather widely spaced boundaries. Such a structure will generally be characterized by high rigidity and by relative immunity to complete failure due to moderate local damage. In general, the more specific the load, however, the greater the liability to injury through occurrence of unforeseen loads.

The use of thin plating perhaps has extended further in ship building (and aircraft construction) than in other branches of engineering. Structures of high weight-efficiency and, at the same time, high adaptability to undetermined loads have been developed, very largely through empirical and evolutional methods. It has more than once occurred that analysis like that used by Mr. Schorer has led to the conclusion that the type of structure evolved by experience has merits which the analysis reveals clearly without being able to carry them much further. Experience has thus rather blindly found the way in advance of analysis. Such cases, however, are exceptional; analysis can proceed systematically and, at least, greatly accelerate progress toward the structure perfectly designed for its function.

[16] *Transactions*, Am. Soc. Mech. Engrs., Vol. 51 (1), 1929 (A. P. M.-51-7), p. 69.
[17] *Transactions*, Am. Soc. Mech. Engrs. (publication pending).

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DISCUSSIONS

NEW JERSEY SEWAGE EXPERIMENTS

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION

Discussion

BY EDMUND B. BESSELIEVRE, M. AM. SOC. C. E.

EDMUND B. BESSELIEVRE,[2] M. AM. SOC. C. E. (by letter).[2a]—The work done by the New Jersey Sewage Experiment Station under the guidance of Dr. Willem Rudolfs has undoubtedly been of great value and aid to the Sanitary Engineering Profession. Work of this character is responsible for much of the progress in sewage treatment that has been made in the ten years since 1920. Instead of groping in the dark and theorizing, intelligently directed experimental and research work can lay the foundation for actual knowledge of reactions and results that make it possible to design sewage plants with assurance of success. Too much cannot be said in favor of this type of research.

It is perhaps to be regretted that all the field work of the New Jersey Station should be carried out at one plant instead of using several plants of different types as field stations. While true comparison of results cannot be obtained from two plants operating on different sewage, nevertheless considerable information on the different conditions and reactions may thus be gained. Except on a relatively small scale, direct comparisons on the same sewage cannot be made and, as a general rule, small-scale experimental work cannot be directly translated into full-sized plant practice without a full study and weighting of all conditions.

Much of the Committee's report is given to discussion of digestion carried on at high temperatures, or "thermophillic" digestion, as it is called. The aim of any method of digestion of fresh organic solids must be to produce a well digested, odorless, quickly drying sludge in the shortest time consistent with true economy. Balanced against the reported short period

NOTE.—This report of the Committee of the Sanitary Engineering Division on New Jersey Sewage Experiments was presented at the meeting of the Sanitary Engineering Division at Norfolk, Va., on April 16, 1931, and published in August, 1931, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[2] San. Engr. and Mgr. Eastern Territory, San. Eng. Div., The Dorr Co., Inc., New York, N. Y.

[2a] Received by the Secretary, October 14, 1931.

achieved in the experiments must be the cost of reproducing the same conditions on a large scale for plants handling large volumes of sludge daily. It is in the large plant that a reduction in time counts, if economy in cost of construction and operation are achieved as well. Against this, however, must be charged the difficulties of insulating large tanks against heat loss and the expense of heating the water to the higher temperatures necessary.

It is pointed out that the volume of gas under "thermophillic" conditions is greater than in digestion at ordinary temperatures. Each pound of organic matter put into a digester is capable of producing a certain quantity of gas. In a short-period digestion method this gas may be discharged in a shorter time, but the total gas on the basis of solids added, will eventually be the same.

Correction of reaction in digestion tanks has been demonstrated so clearly by a great many operating plants in the United States and other countries that it and the value of heating need no defense or discussion. Numerous records are available of plants that have been operated for years without the addition of corrective agents after the initial period of establishment of alkaline balance.

It is stated that liming prevents foaming and scum formation. Scum and foam may also be prevented or eliminated by other means, such as mechanical scum breakers.

The statement is made that managers of great Imhoff plants claim "it is not necessary to stir digesting sludge." Few claims have been made as to the "necessity" of stirring, but the benefits of providing a means for insuring proper contact between the raw solids and digesting sludge have been so clearly shown that it is now recognized as a benefit and economy to provide such means.

The records of the ten years (1922–1931), in the United States, show that the digestion of sludge has been made a reliable process, and this possibility has been the cause for the adoption of the separate digestion method at practically all representative plants built in recent years. Much of the research work which developed the essential features and means for this successful application, has been carried out by or with the assistance of one or two commercial firms whose basic idea was to prove that sludge digestion and other related operations could be made positive and reliable. The use of mechanical units in digestion tanks and in settling tanks has been due to the development of a need for such devices rather than from any attitude of building a machine and then endeavoring to force the opinion that it alone can produce a given result. Surely there would not be to-day a total of more than one hundred and forty digesters in operation, using mechanical means for breaking up scum and maintaining the sludge in a porous condition, if such units had not proved their worth.

The heating plants of many installations of mechanically equipped digesters have burned without interruption for periods of years, indicating a remarkably uniform record of gas production. Many competent engineers have observed that tanks equipped with these thoroughly developed means have little or no scum on them at any time.

Re-circulation of sludge by pumping or other means of discharging the returned material to the top of the tank may serve in some instances to break down scum, due to the impact releasing the gas which is holding the solids at the top. It seems better to provide means that prevent the scum from forming rather than to depend on methods to break it down. One of the criteria of successful operation of a sludge digestion tank is an overflow of displaced liquid of low solid content. It is of no avail if the displaced liquid is high in partly digested organic solids, for it then presents a subsequent disposal problem and is simply transferring the digestion process from one place to another. Re-circulation of large quantities of sludge to the top water will undoubtedly mean that a large proportion of these solids, thrown again into suspension, will pass out of the tank through the overflow. The developed means for preventing scum and keeping bottom sludge in an open porous condition do not move rapidly enough to throw solids into suspension and do not increase the percentage of solids in the overflow. By obtaining a low-moisture sludge from the sedimentation units and adding this daily to the digesters, both the quantity and strength of the displaced liquid will be at a minimum. This relatively small quantity of displaced liquid entails no serious problem in disposal. It may be returned to the raw sewage without effecting the effluent.

The removal of grease, oil, and fibrous and undigestible material from sewage before the sludge is sent to the digesters is of self-evident value as a scum preventive, and also helps to reduce digestion capacity and time. Screens, oil catchers, detritors, and skimmers on sedimentation units have been developed for this purpose.

That the addition of chlorine to raw sewage within limits does not detrimentally affect digestion has been demonstrated at several plants. At Red Bank, N. J., the results of prechlorination have been to increase the removal of solids, prevent scum on the sedimentation units, and a lowered bio-chemical oxygen demand of final effluent. No decrease in gas production in the digester or deterioration of the sludge has been found.

Experience at Birmingham, England, with short-period aeration (1 hour) of settled sewage previous to its application to trickling filters provided with fixed spray nozzles, as compared to unaerated sewage applied to trickling filters with revolving distributors, has developed two decided advantages: First, the absence of flies around the filter; and, second, the absence of odors. With relief from the common fly troubles and from odors, the use of preaeration in large plants could probably be shown to be an economy. In the case of small plants in closely built-up localities it would serve as a protective measure, although not necessarily an economy.

As the use of the activated sludge method of sewage treatment is rapidly gaining, especially for the larger plants, it would seem to be of considerable interest to the profession if the New Jersey Station could carry out some investigations on various phases of that method. Considerable work is being done in the Ruhrverband, Germany, with two-stage aeration and, in several plants, recently inspected by the writer, very good effluents are being

obtained from strong sewage (based on American standards), with a total aeration period of from 2½ to 5 hours. While this method involves an additional sedimentation step between the stages of aeration, the reduction in size of tanks made possible by the shorter period would still make the operation an economy. The benefits of this method as applied to American sewage might be worth while demonstrating. Another phase would be the demonstration of the value of returning the excess activated sludge to the presedimentation tank to demonstrate the advantage of the resultant lower moisture sludge to pass to digestion or filtration.

The New Jersey Sewage Experiment Station, being situated in one of the States which has a great number and variety of sewage plants, has an unexcelled opportunity to make more extended field studies for comparison and should not restrict its efforts to one location. Approached properly, every municipality would be glad to co-operate with the Station in such work, and the funds provided for the operation of the Station should be made available without restriction as to the locale of their use.

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DISCUSSIONS

HIGHWAY CONSTRUCTION MANAGEMENT

Discussion

BY A. E. HOLCOMB, M. AM. SOC. C. E.

A. E. HOLCOMB,[4] M. AM. SOC. C. E. (by letter).[4a]—This paper is both interesting and instructive. The cost of work is frequently affected materially by its location and its timeliness with reference to other work. Unfortunately, economy of construction is seldom the controlling factor in the passing of a bond issue; the volume of noise made by the respective interested parties is usually much more potent. In fact, the psychology of selfish interest is quite often as important a study as the engineering problems involved.

The politician makes the welkin ring with his pre-election grief over the hardships of his constituents and promises to raise them "out of the mud" if he is elected. After election, that locality containing the most votes gets the improvements regardless of whether the work in that section forms a link in a system that will serve the greatest number or, due to location, is more expensive to build than it would be later after another section has been constructed.

Insufficiency of funds and the necessity of giving some relief frequently compel the construction, for instance, of an inadequate drainage system or other project that later must be reconstructed and enlarged at an increased unit cost. Many other reasons of expediency often affect ultimate costs. These are practical considerations that have to be borne in mind. Mr. Allen, however, refers to controllable factors subject to job management.

One of the most frequent decisions that a contractor is called upon to make, is whether to use the equipment he already owns, which is not properly adapted, economically, to the particular job in question, or to purchase new and proper equipment of a size and type to handle the work to the best advantage. In other words, will the increased earnings of new equipment exceed the depreciation and interest charges of the idle equipment? Also, will he

NOTE.—The paper by T. Warren Allen, M. Am. Soc. C. E., was presented at the meeting of the Highway Division, New York, N. Y., January 16, 1930, and published in September, 1931, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[4] Sales Mgr., Koehring Shovel and Crane Div., National Equipment Corporation, Milwaukee, Wis.

[4a] Received by the Secretary October 9, 1931.

be justified financially in depleting his liquid assets at that time by purchasing the needed new equipment? It may be more expedient, or even more profitable, in the end to use his old and less efficient equipment.

On the other hand, because of an inherent, natural reluctance on the part of every one to scrap anything of value, contractors will frequently keep and use obsolescent equipment that is still mechanically in good condition, but "out of step" with newer methods and faster operation. A study of relative output and cost of operation will often show that it would be more profitable to run the obsolescent equipment over the bank—use it as a part of the fill—and buy more modern equipment.

There is often a wide range between the cost to own and the cost to operate. One can well afford to pay considerably more than the average price for equipment, provided the increased earnings within a reasonable length of time will exceed the sum of the increase in first cost, depreciation, and interest charges.

Among the major delays, Mr. Allen mentions 10½% lost time due to repairs to the shovel and other equipment. However, he does not state the nature of the other equipment. The writer's records covering eleven shovels and dragline excavators, over a period of six years, show an average of only 7.8% delay for repairs of sufficient duration to cause a record on the daily report blanks. Such delays are affected by so many things that may or may not be controllable by management that it is impossible, in estimating, to depend on any given range of minimum and maximum repair time figures that would not be excessive. For ordinary and usual conditions, a range of 8 to 10% would be adequate where equipment is maintained in reasonably proper condition. The closer the inspection and the better the upkeep management, the less will be the amount of upkeep delays.

Speaking generally, the more common causes of major delays are unbalanced equipment and lack of foresight in planning operations. For example, in rock work the drilling and blasting are frequently allowed to tie up expensive equipment through the use of the wrong size or kind of drills, lack of sufficient drilling equipment, or improper placing and character of charges. On other work, as Mr. Allen very clearly indicates, one of the greatest causes of loss is the difficulty in balancing and dispatching the hauling equipment.

The real savings, however, are usually to be effected through close personal supervision to keep down the minor, or Class B, delays, since they are largely subject to management control. Mr. Allen refers to improper placing of hauling equipment with relation to the shovel on grading work as an influential factor in the production rate. The writer would suggest in connection with this, the study of the proper width of bench and height of cut to give the most economical results in bank work through deep cuts.

Pre-analysis of jobs, with determination of general plan and layout of work before bidding, supplemented afterward with time studies and distributed costs, is the great need, instead of so much blind and hungry bidding with ever-springing hope and faith in Providence to show the bidder how to cut his costs in two after the award has been made.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SPECIFICATIONS FOR BOUNDARY SURVEYS

PREPARED BY COMMITTEE OF SURVEYING AND MAPPING DIVISION ON BOUNDARY SURVEYS

Discussion

BY J. S. DODDS, M. AM. SOC. C. E.

J. S. DODDS,[6] M. AM. SOC. C. E. (by letter).[6a]—In view of the fact that a large part of the United States is subdivided according to the rectangular system, specific mention of the monuments set in the original Government survey should be made in the specifications.

Those monuments, when recovered, should be perpetuated by means of suitable markers, preferably metallic, and properly referenced to near-by points by not less than four linear measurements. They should also be tied to each other and to the nearest triangulation system monuments of the U. S. Coast and Geodetic Survey by means of triangulation or traverse, or a combination of the two, depending upon local circumstances. The record of the recovery of the original monuments, and the accompanying information relative to their location, should be placed on file as recommended by the Committee. It is a fact, unfortunately, that the records now available of such corners as have been recovered generally fail to show the angular relation of one line to another in the boundaries of the Government subdivisions. Although this information was intended to be shown in the original notes, the discrepancies between the actual conditions and the recorded information are so great that, in all fairness to others who may wish to recover the corners subsequently, the re-surveyor should show the actual lengths of lines and their bearings with reference to one another, as well as their true position geographically, by a system of rectangular co-ordinates referred to the triangulation system of the Coast and Geodetic Survey.

NOTE.—The Specifications for Boundary Surveys, Prepared by the Committee of Surveying and Mapping Division on Boundary Surveys, was published in May, 1931, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1931, by Messrs. George M. Bleekman, G. D. Whitmore, and William W. Michael.

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[6a] Received by the Secretary October 17, 1931.

The writer disagrees with one statement in the first paragraph on "Corrections for Errors." Very frequently, a dispute arises as to the actual location of one of the lines bounding a parcel of land described according to the rectangular system. In such a case, if the monuments at both ends of such a line can be recovered, there certainly is no need to run a closed survey around any of the land partly bounded by that line.

The most important consideration in the entire specification for boundary surveys is that the work be properly monumented. Next to that in importance, of course, is that the distances and angles be accurately measured and recorded. It should be remembered, however, that if the actual monuments can be recovered the property is thereby positively identified.

In the next to the last paragraph of the Committee's report, a very good point is made, that prospective purchasers should be shown the monuments limiting the boundaries of the real property. The writer would amend this to provide that a certified plat showing the location of the tract in question, geographically, and its dimensions both linear and angular, should be acceptable in lieu of an actual examination of the property by the prospective purchaser. In other words, before title to a piece of property passes, a competent survey and plat should be made a part of the permanent title record just as the chain of title is shown in the abstract.

At present, abstractors and lawyers in many parts of the United States do not appreciate the importance of the survey in connection with real property titles and yet, frequently, in the part of the North American Continent covered by a rectangular system of surveys, errors of 10% or more frequently occur in the area of the land being transferred. A survey in any such case would show the area of land being transferred and would eliminate the possibility of much misunderstanding as to the actual property involved in the deal.